
Single and Multivariable Calculus

Late Transcendentals



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This text was initially written by David Guichard. The single variable material in chapters 1–9 is a modification and expansion of notes written by Neal Koblitz at the University of Washington, who generously gave permission to use, modify, and distribute his work. New material has been added, and old material has been modified, so some portions now bear little resemblance to the original.

The book includes some exercises and examples from *Elementary Calculus: An Approach Using Infinitesimals*, by H. Jerome Keisler, available at <http://www.math.wisc.edu/~keisler/calc.html> under a Creative Commons license. In addition, the chapter on differential equations (in the multivariable version) and the section on numerical integration are largely derived from the corresponding portions of Keisler's book. Albert Schueller, Barry Balof, and Mike Wills have contributed additional material.

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I will be glad to receive corrections and suggestions for improvement at guichard@whitman.edu.

*For Kathleen,
without whose encouragement
this book would not have
been written.*

Contents

<hr/>	
1	
Analytic Geometry	15
1.1 Lines	16
1.2 Distance Between Two Points; Circles	21
1.3 Functions	22
1.4 Shifts and Dilations	27
<hr/>	
2	
Instantaneous Rate of Change: The Derivative	31
2.1 The slope of a function	31
2.2 An example	36
2.3 Limits	38
2.4 The Derivative Function	48
2.5 Adjectives For Functions	53

6 Contents

--- 3

Rules for Finding Derivatives	57
3.1 The Power Rule	57
3.2 Linearity of the Derivative	60
3.3 The Product Rule	62
3.4 The Quotient Rule	64
3.5 The Chain Rule	67

--- 4

Trigonometric Functions	73
4.1 Trigonometric Functions	73
4.2 The Derivative of $\sin x$	76
4.3 A hard limit	77
4.4 The Derivative of $\sin x$, continued	80
4.5 Derivatives of the Trigonometric Functions	81
4.6 Implicit Differentiation	82
4.7 Limits revisited	86

--- 5

Curve Sketching	91
5.1 Maxima and Minima	91
5.2 The first derivative test	95
5.3 The second derivative test	96
5.4 Concavity and inflection points	97
5.5 Asymptotes and Other Things to Look For	99

--- 6

Applications of the Derivative	103
6.1 Optimization	103
6.2 Related Rates	115
6.3 Newton's Method	123
6.4 Linear Approximations	127
6.5 The Mean Value Theorem	129

7

Integration	133
7.1 Two examples	133
7.2 The Fundamental Theorem of Calculus	137
7.3 Some Properties of Integrals	144
7.4 Substitution	148

8

Applications of Integration	155
8.1 Area between curves	155
8.2 Distance, Velocity, Acceleration	160
8.3 Volume	163
8.4 Average value of a function	170
8.5 Work	173

9

Transcendental Functions	179
9.1 Inverse functions	179
9.2 The natural logarithm	185
9.3 The exponential function	189
9.4 Other bases	192
9.5 Inverse Trigonometric Functions	196
9.6 Hyperbolic Functions	199

10

Techniques of Integration	205
10.1 Powers of sine and cosine	205
10.2 Trigonometric Substitutions	207
10.3 Integration by Parts	210
10.4 Rational Functions	214
10.5 Numerical Integration	218
10.6 Additional exercises	223

11

More Applications of Integration	225
11.1 Center of Mass	225
11.2 Kinetic energy; improper integrals	231
11.3 Probability	235
11.4 Arc Length	244
11.5 Surface Area	246

12

Polar Coordinates, Parametric Equations	253
12.1 Polar Coordinates	253
12.2 Slopes in polar coordinates	257
12.3 Areas in polar coordinates	259
12.4 Parametric Equations	262
12.5 Calculus with Parametric Equations	265

13

Sequences and Series	269
13.1 Sequences	270
13.2 Series	276
13.3 The Integral Test	280
13.4 Alternating Series	285
13.5 Comparison Tests	287
13.6 Absolute Convergence	290
13.7 The Ratio and Root Tests	291
13.8 Power Series	294
13.9 Calculus with Power Series	297
13.10 Taylor Series	299
13.11 Taylor's Theorem	302
13.12 Additional exercises	308

14

Three Dimensions	311
14.1 The Coordinate System	311
14.2 Vectors	314
14.3 The Dot Product	319
14.4 The Cross Product	325
14.5 Lines and Planes	329
14.6 Other Coordinate Systems	335

15

Vector Functions	341
15.1 Space Curves	341
15.2 Calculus with vector functions	343
15.3 Arc length and curvature	351
15.4 Motion along a curve	357

16

Partial Differentiation	361
16.1 Functions of Several Variables	361
16.2 Limits and Continuity	365
16.3 Partial Differentiation	369
16.4 The Chain Rule	376
16.5 Directional Derivatives	378
16.6 Higher order derivatives	383
16.7 Maxima and minima	385
16.8 Lagrange Multipliers	390

17

Multiple Integration	397
17.1 Volume and Average Height	397
17.2 Double Integrals in Cylindrical Coordinates	407
17.3 Moment and Center of Mass	411
17.4 Surface Area	414
17.5 Triple Integrals	416
17.6 Cylindrical and Spherical Coordinates	419
17.7 Change of Variables	423

18

Vector Calculus	431
18.1 Vector Fields	431
18.2 Line Integrals	433
18.3 The Fundamental Theorem of Line Integrals	437
18.4 Green's Theorem	440
18.5 Divergence and Curl	445
18.6 Vector Functions for Surfaces	448
18.7 Surface Integrals	454
18.8 Stokes's Theorem	458
18.9 The Divergence Theorem	462

19

Differential Equations	467
19.1 First Order Differential Equations	468
19.2 First Order Homogeneous Linear Equations	472
19.3 First Order Linear Equations	475
19.4 Approximation	478
19.5 Second Order Homogeneous Equations	481
19.6 Second Order Linear Equations	485
19.7 Second Order Linear Equations, take two	489

<hr/> A Selected Answers	493
<hr/> B Useful Formulas	519
<hr/> Index	523

Introduction

The emphasis in this course is on problems—doing calculations and story problems. To master problem solving one needs a tremendous amount of practice doing problems. The more problems you do the better you will be at doing them, as patterns will start to emerge in both the problems and in successful approaches to them. You will learn fastest and best if you devote some time to doing problems every day.

Typically the most difficult problems are story problems, since they require some effort before you can begin calculating. Here are some pointers for doing story problems:

1. Carefully read each problem twice before writing anything.
2. Assign letters to quantities that are described only in words; draw a diagram if appropriate.
3. Decide which letters are constants and which are variables. A letter stands for a constant if its value remains the same throughout the problem.
4. Using mathematical notation, write down what you know and then write down what you want to find.
5. Decide what category of problem it is (this might be obvious if the problem comes at the end of a particular chapter, but will not necessarily be so obvious if it comes on an exam covering several chapters).
6. Double check each step as you go along; don't wait until the end to check your work.
7. Use common sense; if an answer is out of the range of practical possibilities, then check your work to see where you went wrong.

14 Introduction

Suggestions for Using This Text

1. Read the example problems carefully, filling in any steps that are left out (ask someone for help if you can't follow the solution to a worked example).
2. Later use the worked examples to study by covering the solutions, and seeing if you can solve the problems on your own.
3. Most exercises have answers in Appendix A; the availability of an answer is marked by “ \Rightarrow ” at the end of the exercise. In the pdf version of the full text, clicking on the arrow will take you to the answer. The answers should be used only as a final check on your work, not as a crutch. Keep in mind that sometimes an answer could be expressed in various ways that are algebraically equivalent, so don't assume that your answer is wrong just because it doesn't have exactly the same form as the answer in the back.
4. A few figures in the pdf and print versions of the book are marked with “(AP)” at the end of the caption. Clicking on this should open a related interactive applet or Sage worksheet in your web browser. Occasionally another link will do the same thing, like this example. (Note to users of a printed text: the words “this example” in the pdf file are blue, and are a link to a Sage worksheet.)