

# Index

**A**  
absolute extremum, 103  
acceleration  
  normal component, 357  
  tangential component, 357  
acceleration vector, 357  
algebraic precedence, 517  
alternating harmonic series, 285  
anti-parallel vectors, 321  
antiderivative, 139  
arc length, 351  
arc length parameterization, 352  
arccosine, 197  
arcsine, 196  
asymptote, 23, 99  
average, 415  
average height, 395

**B**  
bell curve, 238  
binormal, 356, 448  
bounded function, 54

**C**  
cardioid, 254  
Cartesian coordinates, 253, 335  
Cauchy Principal Value, 232  
center of mass, 415  
center of mass, 227  
centroid, 228, 412

of  $\sec^3 x$ , 209  
properties of, 147  
integral sign, 139  
integral test, 282  
integration  
  by parts, 210  
Intermediate Value Theorem, 55  
interval of convergence, 295  
inverse sine, 196  
involute, 264

**J**  
Jacobian, 426  
judicious guessing, 483

**K**  
kinetic energy, 233

**L**  
L'Hôpital's Rule, 87  
Lagrange multipliers, 389  
lateral area of a cone, 114  
law of cosines, 319, 518  
law of sines, 518  
Leibniz notation, 49  
level curve, 363  
level set, 364  
level surface, 364  
limit, 41  
limit at infinity, 87  
limit of a sequence, 271  
line integral, 431  
linear approximation, 127  
linearity of the derivative, 60  
local extremum, 91  
local maximum, 91  
local minimum, 91  
logistic equation, 470  
long division of polynomials, 216

**M**  
Maclaurin series, 299  
mass, 415  
mean, 236, 239  
Mean Value Theorem, 130  
moment, 227, 410, 416

**N**  
Napier's constant, 188

chain rule, 67  
characteristic polynomial, 480  
chord, 32  
circle  
  area, 517  
  circumference, 517  
  equation of, 21, 518  
  unit, 21  
Clairaut's Theorem, 383  
closed paths, 437  
completing the square, 517  
composition of functions, 27, 45, 67  
concave down, 98  
concave up, 98  
cone  
  lateral area, 518  
  surface area, 518  
  volume, 518  
conservative vector field, 436  
continuous, 55, 367  
convergent sequence, 271  
convergent series, 276  
coordinates  
  Cartesian, 253, 335  
  converting rectangular to polar, 254  
  cylindrical, 335  
  polar, 253, 335  
  rectangular, 253, 335  
  spherical, 337  
cosines  
  law of, 319, 518  
critical value, 92  
cross product, 325

Newton, 174  
Newton's law of cooling, 465  
Newton's method, 123–125  
normal, 329, 356  
normal distribution, 243

**O**  
one sided limit, 46  
optimization, 103  
orientable surface, 454  
oriented curve, 439

**P**  
 $p$ -series, 282  
parallel vectors, 321  
parallelogram  
  and vector sum, 315  
  area of, 327, 412, 424  
parametric equations, 262, 332, 342  
partial fractions, 214  
particular solution, 468  
physicists, 116  
point-slope formula, 518  
polar coordinates, 253, 335  
polynomial  
  of two variables, 368  
power function, 57  
power rule, 57  
precedence  
  of algebraic operations, 517  
probability density function, 237  
product rule, 62, 63  
  generalized, 64  
projection, 433  
  scalar, 433  
projection of a vector, 321  
properties of integrals, 147

**Q**  
quadratic formula, 517  
quotient rule, 64

**R**  
radian measure, 73  
radius of convergence, 295  
random variable, 236  
rational function, 66, 214

## 522 Index

cumulative distribution function, 238  
curvature, 354  
curvature formula, 354, 355  
cycloid, 263  
cylinder  
  lateral area, 518  
  surface area, 518  
  volume, 518  
cylindrical coordinates, 417  
cylindrical coordinates, 335

**D**  
definite integral, 143  
del ( $\nabla$ ), 379  
dependent variable, 24  
derivative, 34  
  dot notation, 116  
  Leibniz notation, 49  
difference quotient, 32  
differentiable, 55, 374  
differential, 128  
differential equation, 465  
  first order, 466  
  particular solution, 468  
  separable, 468  
directional derivative, 379  
discrete probability, 235  
discriminant, 384  
displacement vector, 314  
divergence test, 278  
Divergence Theorem, 460  
divergent sequence, 271  
divergent series, 276  
domain, 23  
dot notation, 116  
dot product, 320  
double integral, 397

**E**  
ellipse  
  area, 517  
  equation of, 518  
ellipsoid, 251  
error estimate, 219  
escape velocity, 234  
Euler's Method, 476  
expected value, 236  
exponential distribution, 239  
Extreme Value Theorem, 105

## 524 Index

rectangular coordinates, 253, 335  
related rates, 115  
resonant frequency, 487  
right hand rule, 328  
Rolle's Theorem, 130

**S**  
scalar multiplication, 316  
scalar projection, 433  
separation of variables, 468  
sequence, 270  
  bounded, 274  
  bounded above, 274  
  bounded below, 274  
  convergent, 271  
  decreasing, 274  
  divergent, 271  
  increasing, 274  
  monotonic, 274  
  non-decreasing, 274  
  non-increasing, 274  
  of partial sums, 276  
series, 270  
   $p$ -series, 282  
  absolute convergence, 290  
  alternating harmonic, 285  
  conditional convergence, 290  
  convergent, 276  
  divergent, 276  
  geometric, 276  
  harmonic, 279  
  integral test, 282  
  interval of convergence, 295  
  Maclaurin, 299  
  radius of convergence, 295  
  Taylor, 302  
Simpson's Rule, 221  
sines  
  law of, 518  
slope field, 478  
sphere  
  surface area, 518  
  volume, 518  
spherical coordinates, 337, 417  
spiral of Archimedes, 255  
squeeze theorem, 77  
standard deviation, 241  
standard normal distribution, 239  
standard normal probability density function, 238  
steady state part of solution to d.e., 486

## F

Fermat's Theorem, 92  
flux, 453  
frustum, 247  
Fubini's Theorem, 399  
function, 22  
  bounded, 54  
  differentiable, 55  
  implicit, 82  
  linear, 22  
  of two variables, 368  
  rational, 214  
  unbounded, 54  
function composition, 27  
Fundamental Theorem of Algebra, 95  
Fundamental Theorem of Calculus, 137

## G

Gabriel's horn, 235, 250  
geometric series, 276  
global extremum, 103  
gradient, 379  
greatest integer, 94

## H

harmonic series, 279  
  alternating, 285  
Hooke's Law, 175  
hyperbolic cosine, 199  
hyperbolic sine, 199  
hypercycloid, 264  
hypocycloid, 264

## I

implicit differentiation, 82, 377  
implicit function, 82  
improper integral, 231  
  convergent, 231  
  diverges, 231  
incompressible, 446  
indefinite integral, 143  
independent variable, 24  
inflection point, 98  
initial condition, 466  
initial value problem  
  first order, 466  
integral  
  improper, 231  
  of  $\sec x$ , 209

substitution, 421  
subtend, 74  
sum  
  of vectors, 315  
sum rule, 61  
surface area, 412

## T

tangent line, 32  
Taylor series, 302  
Toricelli's trumpet, 235, 250  
torque, 225  
torus, 250  
transcendental function, 73  
transcendental number, 188  
transient part of solution to d.e., 486  
Trapezoid Rule, 219  
triangle inequality, 43, 314  
trigonometric identities, 518

## U

unbounded function, 54  
undetermined coefficients, 483  
uniform distribution, 238  
uniform probability density function, 238  
unit binormal, 356  
unit circle, 21  
unit normal, 356  
unit vector, 322, 348

## V

variance, 241  
variation of parameters, 473, 487  
vector, 314  
  anti-parallel, 321  
  cross product, 325  
  displacement, 314  
  function, 342  
  normal to a plane, 329  
  parallel, 321  
  projection, 321  
  scalar multiplication, 316  
  sum, 315  
  unit, 322  
vector fields, 429  
velocity, 37  
velocity vector, 357

## W

witch of Agnesi, 66

work, 173