

# A

## Selected Answers

- 1.1.1.  $(2/3)x + (1/3)$
- 1.1.2.  $y = -2x$
- 1.1.3.  $(-2/3)x + (1/3)$
- 1.1.4.  $y = 2x + 2, 2, -1$
- 1.1.5.  $y = -x + 6, 6, 6$
- 1.1.6.  $y = x/2 + 1/2, 1/2, -1$
- 1.1.7.  $y = 3/2, y$ -intercept:  $3/2$ , no  $x$ -intercept
- 1.1.8.  $y = (-2/3)x - 2, -2, -3$
- 1.1.9. yes
- 1.1.10.  $y = 0, y = -2x + 2, y = 2x + 2$
- 1.1.11.  $y = 75t, 164$  minutes
- 1.1.12.  $y = (9/5)x + 32, (-40, -40)$
- 1.1.13.  $y = 0.15x + 10$
- 1.1.14.  $0.03x + 1.2$
- 1.1.15. (a)  $y = \begin{cases} 0 & 0 \leq x < 100 \\ (x/10) - 10 & 100 \leq x \leq 1000 \\ x - 910 & 1000 < x \end{cases}$
- 1.1.16.  $y = \begin{cases} 0.15x & 0 \leq x \leq 19450 \\ 0.28x - 2528.50 & 19450 < x \leq 47050 \\ 0.33x - 4881 & 47050 < x \leq 97620 \end{cases}$
- 1.1.17. (a)  $P = -0.0001x + 2$   
(b)  $x = -10000P + 20000$
- 1.1.18.  $(2/25)x - (16/5)$
- 1.2.1. (a)  $x^2 + y^2 = 9$   
(b)  $(x - 5)^2 + (y - 6)^2 = 9$   
(c)  $(x + 5)^2 + (y + 6)^2 = 9$
- 1.2.2. (a)  $\Delta x = 2, \Delta y = 3, m = 3/2,$   
 $y = (3/2)x - 3, \sqrt{13}$   
(b)  $\Delta x = -1, \Delta y = 3, m = -3,$   
 $y = -3x + 2, \sqrt{10}$   
(c)  $\Delta x = -2, \Delta y = -2, m = 1,$   
 $y = x, \sqrt{8}$
- 1.2.6.  $(x + 2/7)^2 + (y - 41/7)^2 = 1300/49$
- 1.3.1.  $\{x \mid x \geq 3/2\}$
- 1.3.2.  $\{x \mid x \neq -1\}$
- 1.3.3.  $\{x \mid x \neq 1 \text{ and } x \neq -1\}$
- 1.3.4.  $\{x \mid x < 0\}$
- 1.3.5.  $\{x \mid x \in \mathbb{R}\}$ , i.e., all  $x$

- 1.3.6.**  $\{x \mid x \geq 0\}$   
**1.3.7.**  $\{x \mid h - r \leq x \leq h + r\}$   
**1.3.8.**  $\{x \mid x \geq 1 \text{ or } x < 0\}$   
**1.3.9.**  $\{x \mid -1/3 < x < 1/3\}$   
**1.3.10.**  $\{x \mid x \geq 0 \text{ and } x \neq 1\}$   
**1.3.11.**  $\{x \mid x \geq 0 \text{ and } x \neq 1\}$   
**1.3.12.**  $\mathbb{R}$   
**1.3.13.**  $\{x \mid x \geq 3\}, \{x \mid x \geq 0\}$   
**1.3.14.**  $A = x(500 - 2x), \{x \mid 0 \leq x \leq 250\}$   
**1.3.15.**  $V = r(50 - \pi r^2), \{r \mid 0 < r \leq \sqrt{50/\pi}\}$   
**1.3.16.**  $A = 2\pi r^2 + 2000/r, \{r \mid 0 < r < \infty\}$   
**2.1.1.**  $-5, -2.47106145, -2.4067927, -2.400676, -2.4$   
**2.1.2.**  $-4/3, -24/7, 7/24, 3/4$   
**2.1.3.**  $-0.107526881, -0.11074197, -0.1110741, \frac{-1}{3(3 + \Delta x)} \rightarrow \frac{-1}{9}$   
**2.1.4.**  $\frac{3 + 3\Delta x + \Delta x^2}{1 + \Delta x} \rightarrow 3$   
**2.1.5.**  $3.31, 3.003001, 3.0000, 3 + 3\Delta x + \Delta x^2 \rightarrow 3$   
**2.1.6.**  $m$   
**2.2.1.**  $10, 25/2, 20, 15, 25, 35.$   
**2.2.2.**  $5, 4.1, 4.01, 4.001, 4 + \Delta t \rightarrow 4$   
**2.2.3.**  $-10.29, -9.849, -9.8049, -9.8 - 4.9\Delta t \rightarrow -9.8$   
**2.3.1.**  $7$   
**2.3.2.**  $5$   
**2.3.3.**  $0$   
**2.3.4.** undefined  
**2.3.5.**  $1/6$   
**2.3.6.**  $0$   
**2.3.7.**  $3$   
**2.3.8.**  $172$   
**2.3.9.**  $0$   
**2.3.10.**  $2$   
**2.3.11.** does not exist  
**2.3.12.**  $\sqrt{2}$   
**2.3.13.**  $3a^2$   
**2.3.14.**  $512$   
**2.3.15.**  $-4$   
**2.3.16.**  $0$   
**2.3.18.** (a) 8, (b) 6, (c) dne, (d)  $-2$ , (e)  $-1$ , (f) 8, (g) 7, (h) 6, (i) 3, (j)  $-3/2$ , (k) 6, (l) 2  
**2.4.1.**  $-x/\sqrt{169 - x^2}$   
**2.4.2.**  $-9.8t$   
**2.4.3.**  $2x + 1/x^2$   
**2.4.4.**  $2ax + b$   
**2.4.5.**  $3x^2$   
**2.4.8.**  $-2/(2x + 1)^{3/2}$   
**2.4.9.**  $5/(t + 2)^2$   
**2.4.10.**  $y = -13x + 17$   
**2.4.11.**  $-8$   
**3.1.1.**  $100x^{99}$   
**3.1.2.**  $-100x^{-101}$   
**3.1.3.**  $-5x^{-6}$   
**3.1.4.**  $\pi x^{\pi-1}$   
**3.1.5.**  $(3/4)x^{-1/4}$   
**3.1.6.**  $-(9/7)x^{-16/7}$   
**3.2.1.**  $15x^2 + 24x$   
**3.2.2.**  $-20x^4 + 6x + 10/x^3$   
**3.2.3.**  $-30x + 25$   
**3.2.4.**  $6x^2 + 2x - 8$

- 3.2.5.**  $3x^2 + 6x - 1$   
**3.2.6.**  $9x^2 - x/\sqrt{625 - x^2}$   
**3.2.7.**  $y = 13x/4 + 5$   
**3.2.8.**  $y = 24x - 48 - \pi^3$   
**3.2.9.**  $-49t/5 + 5, -49/5$   
**3.2.11.**  $\sum_{k=1}^n ka_k x^{k-1}$   
**3.2.12.**  $x^3/16 - 3x/4 + 4$   
**3.3.1.**  $3x^2(x^3 - 5x + 10) + x^3(3x^2 - 5)$   
**3.3.2.**  $(x^2 + 5x - 3)(5x^4 - 18x^2 + 6x - 7) + (2x + 5)(x^5 - 6x^3 + 3x^2 - 7x + 1)$   
**3.3.3.**  $\frac{\sqrt{625 - x^2}}{2\sqrt{x}} - \frac{x\sqrt{x}}{\sqrt{625 - x^2}}$   
**3.3.4.**  $\frac{-1}{x^{19}\sqrt{625 - x^2}} - \frac{20\sqrt{625 - x^2}}{x^{21}}$   
**3.3.5.**  $f' = 4(2x - 3), y = 4x - 7$   
**3.4.1.**  $\frac{3x^2}{x^3 - 5x + 10} - \frac{x^3(3x^2 - 5)}{(x^3 - 5x + 10)^2}$   
**3.4.2.**  $\frac{2x + 5}{x^5 - 6x^3 + 3x^2 - 7x + 1} - \frac{(x^2 + 5x - 3)(5x^4 - 18x^2 + 6x - 7)}{(x^5 - 6x^3 + 3x^2 - 7x + 1)^2}$   
**3.4.3.**  $\frac{1}{2\sqrt{x}\sqrt{625 - x^2}} + \frac{x^{3/2}}{(625 - x^2)^{3/2}}$   
**3.4.4.**  $\frac{-1}{x^{19}\sqrt{625 - x^2}} - \frac{20\sqrt{625 - x^2}}{x^{21}}$   
**3.4.5.**  $y = 17x/4 - 41/4$   
**3.4.6.**  $y = 11x/16 - 15/16$   
**3.4.8.**  $y = 19/169 - 5x/338$   
**3.4.9.**  $13/18$   
**3.5.1.**  $4x^3 - 9x^2 + x + 7$   
**3.5.2.**  $3x^2 - 4x + 2/\sqrt{x}$   
**3.5.3.**  $6(x^2 + 1)^2x$   
**3.5.4.**  $\sqrt{169 - x^2} - x^2/\sqrt{169 - x^2}$   
**3.5.5.**  $(2x - 4)\sqrt{25 - x^2} - (x^2 - 4x + 5)x/\sqrt{25 - x^2}$   
**3.5.6.**  $-x/\sqrt{r^2 - x^2}$   
**3.5.7.**  $2x^3/\sqrt{1 + x^4}$   
**3.5.8.**  $\frac{1}{4\sqrt{x}(5 - \sqrt{x})^{3/2}}$   
**3.5.9.**  $6 + 18x$   
**3.5.10.**  $\frac{2x + 1}{1 - x} + \frac{x^2 + x + 1}{(1 - x)^2}$   
**3.5.11.**  $-1/\sqrt{25 - x^2} - \sqrt{25 - x^2}/x^2$   
**3.5.12.**  $\frac{1}{2} \left( \frac{-169}{x^2} - 1 \right) / \sqrt{\frac{169}{x} - x}$   
**3.5.13.**  $\frac{3x^2 - 2x + 1/x^2}{2\sqrt{x^3 - x^2 - (1/x)}}$   
**3.5.14.**  $\frac{300x}{(100 - x^2)^{5/2}}$   
**3.5.15.**  $\frac{1 + 3x^2}{3(x + x^3)^{2/3}}$   
**3.5.16.**  $\left( 4x(x^2 + 1) + \frac{4x^3 + 4x}{2\sqrt{1 + (x^2 + 1)^2}} \right) / 2\sqrt{(x^2 + 1)^2 + \sqrt{1 + (x^2 + 1)^2}}$   
**3.5.17.**  $5(x + 8)^4$   
**3.5.18.**  $-3(4 - x)^2$   
**3.5.19.**  $6x(x^2 + 5)^2$   
**3.5.20.**  $-12x(6 - 2x^2)^2$   
**3.5.21.**  $24x^2(1 - 4x^3)^{-3}$   
**3.5.22.**  $5 + 5/x^2$   
**3.5.23.**  $-8(4x - 1)(2x^2 - x + 3)^{-3}$   
**3.5.24.**  $1/(x + 1)^2$   
**3.5.25.**  $3(8x - 2)/(4x^2 - 2x + 1)^2$   
**3.5.26.**  $-3x^2 + 5x - 1$   
**3.5.27.**  $6x(2x - 4)^3 + 6(3x^2 + 1)(2x - 4)^2$

3.5.28.  $-2/(x-1)^2$

3.5.29.  $4x/(x^2+1)^2$

3.5.30.  $(x^2-6x+7)/(x-3)^2$

3.5.31.  $-5/(3x-4)^2$

3.5.32.  $60x^4+72x^3+18x^2+18x-6$

3.5.33.  $(5-4x)/((2x+1)^2(x-3)^2)$

3.5.34.  $1/(2(2+3x)^2)$

3.5.35.  $56x^6+72x^5+110x^4+100x^3+60x^2+28x+6$

3.5.36.  $y=23x/96-29/96$

3.5.37.  $y=3-2x/3$

3.5.38.  $y=13x/2-23/2$

3.5.39.  $y=2x-11$

3.5.40.  $y = \frac{20+2\sqrt{5}}{5\sqrt{4+\sqrt{5}}}x + \frac{3\sqrt{5}}{5\sqrt{4+\sqrt{5}}}$

4.1.1.  $2n\pi - \pi/2$ , any integer  $n$

4.1.2.  $n\pi \pm \pi/6$ , any integer  $n$

4.1.3.  $(\sqrt{2} + \sqrt{6})/4$

4.1.4.  $-(1 + \sqrt{3})/(1 - \sqrt{3}) = 2 + \sqrt{3}$

4.1.11.  $t = \pi/2$

4.3.1. 5

4.3.2.  $7/2$

4.3.3.  $3/4$

4.3.4. 1

4.3.5.  $-\sqrt{2}/2$

4.3.6. 7

4.3.7. 2

4.4.1.  $\sin(\sqrt{x})\cos(\sqrt{x})/\sqrt{x}$

4.4.2.  $\frac{\sin x}{2\sqrt{x}} + \sqrt{x}\cos x$

4.4.3.  $-\frac{\cos x}{\sin^2 x}$

4.4.4.  $\frac{(2x+1)\sin x - (x^2+x)\cos x}{\sin^2 x}$

4.4.5.  $\frac{-\sin x \cos x}{\sqrt{1-\sin^2 x}}$

4.5.1.  $\cos^2 x - \sin^2 x$

4.5.2.  $-\sin x \cos(\cos x)$

4.5.3.  $\frac{\tan x + x \sec^2 x}{2\sqrt{x} \tan x}$

4.5.4.  $\frac{\sec^2 x(1+\sin x) - \tan x \cos x}{(1+\sin x)^2}$

4.5.5.  $-\csc^2 x$

4.5.6.  $-\csc x \cot x$

4.5.7.  $3x^2 \sin(23x^2) + 46x^4 \cos(23x^2)$

4.5.8. 0

4.5.9.  $-6 \cos(\cos(6x)) \sin(6x)$

4.5.10.  $\sin \theta / (\cos \theta + 1)^2$

4.5.11.  $5t^4 \cos(6t) - 6t^5 \sin(6t)$

4.5.12.  $3t^2(\sin(3t) + t \cos(3t)) / \cos(2t) + 2t^3 \sin(3t) \sin(2t) / \cos^2(2t)$

4.5.13.  $n\pi/2$ , any integer  $n$

4.5.14.  $\pi/2 + n\pi$ , any integer  $n$

4.5.15.  $\sqrt{3}x/2 + 3/4 - \sqrt{3}\pi/6$

4.5.16.  $8\sqrt{3}x + 4 - 8\sqrt{3}\pi/3$

4.5.17.  $3\sqrt{3}x/2 - \sqrt{3}\pi/4$

4.5.18.  $\pi/6 + 2n\pi, 5\pi/6 + 2n\pi$ , any integer  $n$

4.6.1.  $x/y$

4.6.2.  $-(2x+y)/(x+2y)$

4.6.3.  $(2xy - 3x^2 - y^2)/(2xy - 3y^2 - x^2)$

4.6.4.  $\sin(x)\sin(y)/(\cos(x)\cos(y))$

4.6.5.  $-\sqrt{y}/\sqrt{x}$

4.6.6.  $(y \sec^2(x/y) - y^2)/(x \sec^2(x/y) + y^2)$

4.6.7.  $(y - \cos(x+y))/(\cos(x+y) - x)$

4.6.8.  $-y^2/x^2$

- 4.6.9.** 1  
**4.6.11.**  $y = 2x \pm 6$   
**4.6.12.**  $y = x/2 \pm 3$   
**4.6.13.**  $(\sqrt{3}, 2\sqrt{3}), (-\sqrt{3}, -2\sqrt{3}),$   
 $(2\sqrt{3}, \sqrt{3}), (-2\sqrt{3}, -\sqrt{3})$   
**4.6.14.**  $y = 7x/\sqrt{3} - 8/\sqrt{3}$   
**4.6.15.**  $y = (-y_1^{1/3}x + y_1^{1/3}x_1 + x_1^{1/3}y_1)/x_1^{1/3}$   
**4.6.16.**  $(y - y_1)/(x - x_1) = (2x_1^3 + 2x_1y_1^2 -$   
 $x_1)/(2y_1^3 + 2y_1x_1^2 + y_1)$   
**4.7.1.** 0  
**4.7.2.** 1  
**4.7.3.**  $1/6$   
**4.7.4.**  $-\infty$   
**4.7.5.**  $1/16$   
**4.7.6.**  $1/3$   
**4.7.7.** 0  
**4.7.8.**  $3/2$   
**4.7.9.**  $-1/4$   
**4.7.10.**  $-3$   
**4.7.11.**  $1/2$   
**4.7.12.** 0  
**4.7.13.**  $-1$   
**4.7.14.**  $-1/2$   
**4.7.15.** 5  
**4.7.16.**  $\infty$   
**4.7.17.**  $\infty$   
**4.7.18.**  $2/7$   
**4.7.19.** 2  
**4.7.20.**  $-\infty$   
**4.7.21.** 1  
**4.7.22.** 0  
**4.7.23.**  $\infty$   
**4.7.24.**  $1/2$   
**4.7.25.** 0  
**4.7.26.**  $1/2$   
**4.7.27.** 5  
**4.7.28.**  $2\sqrt{2}$   
**4.7.29.**  $-1/2$   
**4.7.30.** 2  
**4.7.31.** 0  
**4.7.32.**  $\infty$   
**4.7.33.** 0  
**4.7.34.**  $3/2$   
**4.7.35.**  $\infty$   
**4.7.36.** 5  
**4.7.37.**  $-1/2$   
**4.7.38.** does not exist  
**4.7.39.**  $\infty$   
**4.7.40.**  $y = 1$  and  $y = -1$   
**5.1.1.** min at  $x = 1/2$   
**5.1.2.** min at  $x = -1$ , max at  $x = 1$   
**5.1.3.** max at  $x = 2$ , min at  $x = 4$   
**5.1.4.** min at  $x = \pm 1$ , max at  $x = 0$ .  
**5.1.5.** min at  $x = 1$   
**5.1.6.** none  
**5.1.7.** none  
**5.1.8.** min at  $x = 7\pi/12 + k\pi$ , max at  
 $x = -\pi/12 + k\pi$ , for integer  $k$ .  
**5.1.9.** none  
**5.1.10.** local max at  $x = 5$   
**5.1.11.** local min at  $x = 49$   
**5.1.12.** local min at  $x = 0$   
**5.1.15.** one  
**5.2.1.** min at  $x = 1/2$

- 5.2.2. min at  $x = -1$ , max at  $x = 1$
- 5.2.3. max at  $x = 2$ , min at  $x = 4$
- 5.2.4. min at  $x = \pm 1$ , max at  $x = 0$ .
- 5.2.5. min at  $x = 1$
- 5.2.6. none
- 5.2.7. none
- 5.2.8. min at  $x = 7\pi/12 + k\pi$ , max at  $x = -\pi/12 + k\pi$ , for integer  $k$ .
- 5.2.9. none
- 5.2.10. max at  $x = 0$ , min at  $x = \pm 11$
- 5.2.11. min at  $x = -3/2$ , neither at  $x = 0$
- 5.2.13. min at  $n\pi$ , max at  $\pi/2 + n\pi$
- 5.2.14. min at  $2n\pi$ , max at  $(2n + 1)\pi$
- 5.2.15. min at  $\pi/2 + 2n\pi$ , max at  $3\pi/2 + 2n\pi$
- 5.3.1. min at  $x = 1/2$
- 5.3.2. min at  $x = -1$ , max at  $x = 1$
- 5.3.3. max at  $x = 2$ , min at  $x = 4$
- 5.3.4. min at  $x = \pm 1$ , max at  $x = 0$ .
- 5.3.5. min at  $x = 1$
- 5.3.6. none
- 5.3.7. none
- 5.3.8. min at  $x = 7\pi/12 + n\pi$ , max at  $x = -\pi/12 + n\pi$ , for integer  $n$ .
- 5.3.9. max at  $x = 63/64$
- 5.3.10. max at  $x = 7$
- 5.3.11. max at  $-5^{-1/4}$ , min at  $5^{-1/4}$
- 5.3.12. none
- 5.3.13. max at  $-1$ , min at  $1$
- 5.3.14. min at  $2^{-1/3}$
- 5.3.15. none
- 5.3.16. min at  $n\pi$
- 5.3.17. max at  $n\pi$ , min at  $\pi/2 + n\pi$
- 5.3.18. max at  $\pi/2 + 2n\pi$ , min at  $3\pi/2 + 2n\pi$
- 5.4.1. concave up everywhere
- 5.4.2. concave up when  $x < 0$ , concave down when  $x > 0$
- 5.4.3. concave down when  $x < 3$ , concave up when  $x > 3$
- 5.4.4. concave up when  $x < -1/\sqrt{3}$  or  $x > 1/\sqrt{3}$ , concave down when  $-1/\sqrt{3} < x < 1/\sqrt{3}$
- 5.4.5. concave up when  $x < 0$  or  $x > 2/3$ , concave down when  $0 < x < 2/3$
- 5.4.6. concave up when  $x < 0$ , concave down when  $x > 0$
- 5.4.7. concave up when  $x < -1$  or  $x > 1$ , concave down when  $-1 < x < 0$  or  $0 < x < 1$
- 5.4.8. concave down on  $((8n - 1)\pi/4, (8n + 3)\pi/4)$ , concave up on  $((8n + 3)\pi/4, (8n + 7)\pi/4)$ , for integer  $n$
- 5.4.9. concave down everywhere
- 5.4.10. concave up on  $(-\infty, (21 - \sqrt{497})/4)$  and  $(21 + \sqrt{497})/4, \infty)$
- 5.4.11. concave up on  $(0, \infty)$
- 5.4.12. concave down on  $(2n\pi/3, (2n + 1)\pi/3)$
- 5.4.13. concave up on  $(0, \infty)$
- 5.4.14. concave up on  $(-\infty, -1)$  and  $(0, \infty)$
- 5.4.15. concave down everywhere
- 5.4.16. concave up everywhere
- 5.4.17. concave up on  $(\pi/4 + n\pi, 3\pi/4 + n\pi)$
- 5.4.18. inflection points at  $n\pi$ ,  $\pm \arcsin(\sqrt{2/3}) + n\pi$
- 5.4.19. up/incr:  $(3, \infty)$ , up/decr:  $(-\infty, 0)$ ,  $(2, 3)$ , down/decr:  $(0, 2)$

- 6.1.1.** max at  $(2, 5)$ , min at  $(0, 1)$   
**6.1.2.**  $25 \times 25$   
**6.1.3.**  $P/4 \times P/4$   
**6.1.4.**  $w = l = 2 \cdot 5^{2/3}$ ,  $h = 5^{2/3}$ ,  $h/w = 1/2$   
**6.1.5.**  $\sqrt[3]{100} \times \sqrt[3]{100} \times 2\sqrt[3]{100}$ ,  $h/s = 2$   
**6.1.6.**  $w = l = 2^{1/3}V^{1/3}$ ,  $h = V^{1/3}/2^{2/3}$ ,  $h/w = 1/2$   
**6.1.7.** 1250 square feet  
**6.1.8.**  $l^2/8$  square feet  
**6.1.9.** \$5000  
**6.1.10.** 100  
**6.1.11.**  $r^2$   
**6.1.12.**  $h/r = 2$   
**6.1.13.**  $h/r = 2$   
**6.1.14.**  $r = 5$ ,  $h = 40/\pi$ ,  $h/r = 8/\pi$   
**6.1.15.**  $8/\pi$   
**6.1.16.**  $4/27$   
**6.1.17.** Go direct from  $A$  to  $D$ .  
**6.1.18.** (a) 2, (b)  $7/2$   
**6.1.19.**  $\frac{\sqrt{3}}{6} \times \frac{\sqrt{3}}{6} + \frac{1}{2} \times \frac{1}{4} - \frac{\sqrt{3}}{12}$   
**6.1.20.** (a)  $a/6$ , (b)  $(a + b - \sqrt{a^2 - ab + b^2})/6$   
**6.1.21.** 1.5 meters wide by 1.25 meters tall  
**6.1.22.** If  $k \leq 2/\pi$  the ratio is  $(2 - k\pi)/4$ ; if  $k \geq 2/\pi$ , the ratio is zero: the window should be semicircular with no rectangular part.  
**6.1.23.**  $a/b$   
**6.1.24.**  $w = 2r/\sqrt{3}$ ,  $h = 2\sqrt{2}r/\sqrt{3}$   
**6.1.25.**  $1/\sqrt{3} \approx 58\%$   
**6.1.26.**  $18 \times 18 \times 36$   
**6.1.27.**  $r = 5/(2\pi)^{1/3} \approx 2.7$  cm,  
 $h = 5 \cdot 2^{5/3}/\pi^{1/3} = 4r \approx 10.8$  cm  
**6.1.28.**  $h = \frac{750}{\pi} \left( \frac{2\pi^2}{750^2} \right)^{1/3}$ ,  $r = \left( \frac{750^2}{2\pi^2} \right)^{1/6}$   
**6.1.29.**  $h/r = \sqrt{2}$   
**6.1.30.** The ratio of the volume of the sphere to the volume of the cone is  $1033/4096 + 33/4096\sqrt{17} \approx 0.2854$ , so the cone occupies approximately 28.54% of the sphere.  
**6.1.31.**  $P$  should be at distance  $c\sqrt[3]{a}/(\sqrt[3]{a} + \sqrt[3]{b})$  from charge  $A$ .  
**6.1.32.**  $1/2$   
**6.1.33.** \$7000  
**6.1.34.** There is a critical point when  $\sin \theta_1/v_1 = \sin \theta_2/v_2$ , and the second derivative is positive, so there is a minimum at the critical point.  
**6.2.1.**  $1/(16\pi)$  cm/s  
**6.2.2.**  $3/(1000\pi)$  meters/second  
**6.2.3.**  $1/4$  m/s  
**6.2.4.**  $-6/25$  m/s  
**6.2.5.**  $80\pi$  mi/min  
**6.2.6.**  $3\sqrt{5}$  ft/s  
**6.2.7.**  $20/(3\pi)$  cm/s  
**6.2.8.**  $13/20$  ft/s  
**6.2.9.**  $5\sqrt{10}/2$  m/s  
**6.2.10.**  $75/64$  m/min  
**6.2.11.**  $145\pi/72$  m/s  
**6.2.12.**  $25\pi/144$  m/min

- 6.2.13.  $\pi\sqrt{2}/36$  ft<sup>3</sup>/s
- 6.2.14. tip: 6 ft/s, length: 5/2 ft/s
- 6.2.15. tip: 20/11 m/s, length: 9/11 m/s
- 6.2.16.  $380/\sqrt{3} - 150 \approx 69.4$  mph
- 6.2.17.  $500/\sqrt{3} - 200 \approx 88.7$  km/hr
- 6.2.18. 18 m/s
- 6.2.19.  $136\sqrt{475}/19 \approx 156$  km/hr
- 6.2.20. -50 m/s
- 6.2.21. 68 m/s
- 6.2.22.  $3800/\sqrt{329} \approx 210$  km/hr
- 6.2.23.  $820/\sqrt{329} + 150\sqrt{57}/\sqrt{47} \approx 210$  km/hr
- 6.2.24. 4000/49 m/s
- 6.2.25. (a)  $x = a \cos \theta - a \sin \theta \cot(\theta + \beta) = a \sin \beta / \sin(\theta + \beta)$ , (c)  $\dot{x} \approx 3.79$  cm/s
- 6.3.1.  $x_3 = 1.475773162$
- 6.3.2. 2.15
- 6.3.3. 3.36
- 6.3.4. 2.19 or 1.26
- 6.4.1.  $\Delta y = 65/16$ ,  $dy = 2$
- 6.4.2.  $\Delta y = \sqrt{11/10} - 1$ ,  $dy = 0.05$
- 6.4.3.  $\Delta y = \sin(\pi/50)$ ,  $dy = \pi/50$
- 6.4.4.  $dV = 8\pi/25$
- 6.5.1.  $c = 1/2$
- 6.5.2.  $c = \sqrt{18} - 2$
- 6.5.6.  $x^3/3 + 47x^2/2 - 5x + k$
- 6.5.7.  $-\cos(2x)/2 + k$
- 7.1.1. 10
- 7.1.2. 35/3
- 7.1.3.  $x^2$
- 7.1.4.  $2x^2$
- 7.1.5.  $2x^2 - 8$
- 7.1.6.  $2b^2 - 2a^2$
- 7.1.7. 4 rectangles:  $41/4 = 10.25$ ,  
8 rectangles:  $183/16 = 11.4375$
- 7.1.8. 23/4
- 7.2.1.  $(16/3)x^{3/2} + C$
- 7.2.2.  $t^3 + t + C$
- 7.2.3.  $8\sqrt{x} + C$
- 7.2.4.  $-2/z + C$
- 7.2.5.  $(5x + 1)^3/15 + C$
- 7.2.6.  $(x - 6)^3/3 + C$
- 7.2.7.  $2x^{5/2}/5 + C$
- 7.2.8.  $-4/\sqrt{x} + C$
- 7.2.9.  $4t - t^2 + C$ ,  $t < 2$ ;  $t^2 - 4t + 8 + C$ ,  
 $t \geq 2$
- 7.2.10. 87/2
- 7.2.11. 2
- 7.2.12.  $3^4/4$
- 7.2.13.  $2^6/6 - 1/6$
- 7.2.14.  $x^2 - 3x$
- 7.2.15.  $2x(x^4 - 3x^2)$
- 7.2.16.  $\tan(x^2)$
- 7.2.17.  $2x \tan(x^4)$
- 7.3.1. It rises until  $t = 100/49$ , then falls. The position of the object at time  $t$  is  $s(t) = -4.9t^2 + 20t + k$ . The net distance traveled is  $-45/2$ , that is, it ends up 45/2 meters below where it started. The total distance traveled is 6205/98 meters.
- 7.3.2.  $\int_0^{2\pi} \sin t \, dt = 0$
- 7.3.3. net:  $2\pi$ , total:  $2\pi/3 + 4\sqrt{3}$
- 7.3.4. 8
- 7.3.5. 17/3



- 7.3.6.**  $A = 18, B = 44/3, C = 10/3$   
**7.4.1.**  $-(1-t)^{10}/10 + C$   
**7.4.2.**  $x^5/5 + 2x^3/3 + x + C$   
**7.4.3.**  $(x^2 + 1)^{101}/202 + C$   
**7.4.4.**  $-3(1-5t)^{2/3}/10 + C$   
**7.4.5.**  $(\sin^4 x)/4 + C$   
**7.4.6.**  $-(100-x^2)^{3/2}/3 + C$   
**7.4.7.**  $-2\sqrt{1-x^3}/3 + C$   
**7.4.8.**  $\sin(\sin \pi t)/\pi + C$   
**7.4.9.**  $1/(2 \cos^2 x) = (1/2) \sec^2 x + C$   
**7.4.10.**  $-\ln |\cos x| + C$   
**7.4.11.** 0  
**7.4.12.**  $\tan^2(x)/2 + C$   
**7.4.13.**  $1/4$   
**7.4.14.**  $-\cos(\tan x) + C$   
**7.4.15.**  $1/10$   
**7.4.16.**  $\sqrt{3}/4$   
**7.4.17.**  $(27/8)(x^2 - 7)^{8/9}$   
**7.4.18.**  $-(3^7 + 1)/14$   
**7.4.19.** 0  
**7.4.20.**  $f(x)^2/2$   
**8.1.1.**  $8\sqrt{2}/15$   
**8.1.2.**  $1/12$   
**8.1.3.**  $9/2$   
**8.1.4.**  $4/3$   
**8.1.5.**  $2/3 - 2/\pi$   
**8.1.6.**  $3/\pi - 3\sqrt{3}/(2\pi) - 1/8$   
**8.1.7.**  $1/3$   
**8.1.8.**  $10\sqrt{5}/3 - 6$   
**8.1.9.**  $500/3$   
**8.1.10.** 2  
**8.1.11.**  $1/5$   
**8.1.12.**  $1/6$   
**8.2.1.**  $1/\pi, 5/\pi$   
**8.2.2.** 0, 245  
**8.2.3.** 20, 28  
**8.2.4.**  $(3-\pi)/(2\pi), (18-12\sqrt{3}+\pi)/(4\pi)$   
**8.2.5.** 10/49 meters, 20/49 seconds  
**8.2.6.** 45/98 meters, 30/49 seconds  
**8.2.7.** 25000/49 meters, 1000/49 seconds  
**8.2.8.**  $s(t) = \cos t, v(t) = -\sin t$ ,  
 maximum distance is 1,  
 maximum speed is 1  
**8.2.9.**  $s(t) = -\sin(\pi t)/\pi^2 + t/\pi$ ,  
 $v(t) = -\cos(\pi t)/\pi + 1/\pi$ ,  
 maximum speed is  $2/\pi$   
**8.2.10.**  $s(t) = t^2/2 - \sin(\pi t)/\pi^2 + t/\pi$ ,  
 $v(t) = t - \cos(\pi t)/\pi + 1/\pi$   
**8.2.11.**  $s(t) = t^2/2 + \sin(\pi t)/\pi^2 - t/\pi$ ,  
 $v(t) = t + \cos(\pi t)/\pi - 1/\pi$   
**8.3.5.**  $8\pi/3$   
**8.3.6.**  $\pi/30$   
**8.3.7.**  $\pi(\pi/2 - 1)$   
**8.3.8.** (a)  $114\pi/5$  (b)  $74\pi/5$  (c)  $20\pi$   
 (d)  $4\pi$   
**8.3.9.**  $16\pi, 24\pi$   
**8.3.11.**  $\pi h^2(3r-h)/3$   
**8.3.13.**  $2\pi$   
**8.4.1.**  $2/\pi; 2/\pi; 0$   
**8.4.2.**  $4/3$   
**8.4.3.**  $1/A$   
**8.4.4.**  $\pi/4$   
**8.4.5.**  $-1/3, 1$   
**8.4.6.**  $-4\sqrt{1224}$  ft/s;  $-8\sqrt{1224}$  ft/s

- 8.5.1.  $\approx 5,305,028,516$  N-m  
 8.5.2.  $\approx 4,457,854,041$  N-m  
 8.5.3.  $367,500\pi$  N-m  
 8.5.4.  $49000\pi + 196000/3$  N-m  
 8.5.5.  $2450\pi$  N-m  
 8.5.6.  $0.05$  N-m  
 8.5.7.  $6/5$  N-m  
 8.5.8.  $3920$  N-m  
 8.5.9.  $23520$  N-m  
 8.5.10.  $12740$  N-m  
 9.5.3.  $\frac{-1}{1+x^2}$   
 9.5.5.  $\frac{2x}{\sqrt{1-x^4}}$   
 9.5.6.  $\frac{e^x}{1+e^{2x}}$   
 9.5.7.  $-3x^2 \cos(x^3)/\sqrt{1-\sin^2(x^3)}$   
 9.5.8.  $\frac{2}{(\arcsin x)\sqrt{1-x^2}}$   
 9.5.9.  $-e^x/\sqrt{1-e^{2x}}$   
 9.5.10.  $0$   
 9.5.11.  $\frac{(1+\ln x)x^x}{\ln 5(1+x^{2x})\arctan(x^x)}$   
 10.1.1.  $x/2 - \sin(2x)/4 + C$   
 10.1.2.  $-\cos x + (\cos^3 x)/3 + C$   
 10.1.3.  $3x/8 - (\sin 2x)/4 + (\sin 4x)/32 + C$   
 10.1.4.  $(\cos^5 x)/5 - (\cos^3 x)/3 + C$   
 10.1.5.  $\sin x - (\sin^3 x)/3 + C$   
 10.1.6.  $x/8 - (\sin 4x)/32 + C$   
 10.1.7.  $(\sin^3 x)/3 - (\sin^5 x)/5 + C$   
 10.1.8.  $-2(\cos x)^{5/2}/5 + C$   
 10.1.9.  $\tan x - \cot x + C$   
 10.1.10.  $(\sec^3 x)/3 - \sec x + C$   
 10.2.1.  $-\ln |\csc x + \cot x| + C$   
 10.2.2.  $-\csc x \cot x/2 - (1/2)\ln |\csc x + \cot x| + C$   
 10.2.3.  $x\sqrt{x^2-1}/2 - \ln |x + \sqrt{x^2-1}|/2 + C$   
 10.2.4.  $x\sqrt{9+4x^2}/2 + (9/4)\ln |2x + \sqrt{9+4x^2}| + C$   
 10.2.5.  $-(1-x^2)^{3/2}/3 + C$   
 10.2.6.  $\arcsin(x)/8 - \sin(4\arcsin x)/32 + C$   
 10.2.7.  $\ln |x + \sqrt{1+x^2}| + C$   
 10.2.8.  $(x+1)\sqrt{x^2+2x}/2 - \ln |x+1 + \sqrt{x^2+2x}|/2 + C$   
 10.2.9.  $-\arctan x - 1/x + C$   
 10.2.10.  $2\arcsin(x/2) - x\sqrt{4-x^2}/2 + C$   
 10.2.11.  $\arcsin(\sqrt{x}) - \sqrt{x}\sqrt{1-x} + C$   
 10.2.12.  $(2x^2+1)\sqrt{4x^2-1}/24 + C$   
 10.3.1.  $\cos x + x \sin x + C$   
 10.3.2.  $x^2 \sin x - 2 \sin x + 2x \cos x + C$   
 10.3.3.  $(x-1)e^x + C$   
 10.3.4.  $(1/2)e^{x^2} + C$   
 10.3.5.  $(x/2) - \sin(2x)/4 + C = (x/2) - (\sin x \cos x)/2 + C$   
 10.3.6.  $x \ln x - x + C$   
 10.3.7.  $(x^2 \arctan x + \arctan x - x)/2 + C$   
 10.3.8.  $-x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + C$   
 10.3.9.  $x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6 \cos x + C$   
 10.3.10.  $x^2/4 - (\cos^2 x)/4 - (x \sin x \cos x)/2 + C$   
 10.3.11.  $x/4 - (x \cos^2 x)/2 + (\cos x \sin x)/4 + C$   
 10.3.12.  $x \arctan(\sqrt{x}) + \arctan(\sqrt{x}) - \sqrt{x} + C$

10.3.13.  $2 \sin(\sqrt{x}) - 2\sqrt{x} \cos(\sqrt{x}) + C$

10.3.14.  $\sec x \csc x - 2 \cot x + C$

10.4.1.  $-\ln|x-2|/4 + \ln|x+2|/4 + C$

10.4.2.  $-x^3/3 - 4x - 4 \ln|x-2| + 4 \ln|x+2| + C$

10.4.3.  $-1/(x+5) + C$

10.4.4.  $-x - \ln|x-2| + \ln|x+2| + C$

10.4.5.  $-4x + x^3/3 + 8 \arctan(x/2) + C$

10.4.6.  $(1/2) \arctan(x/2 + 5/2) + C$

10.4.7.  $x^2/2 - 2 \ln(4+x^2) + C$

10.4.8.  $(1/4) \ln|x+3| - (1/4) \ln|x+7| + C$

10.4.9.  $(1/5) \ln|2x-3| - (1/5) \ln|1+x| + C$

10.4.10.  $(1/3) \ln|x| - (1/3) \ln|x+3| + C$

10.5.1. T,S:  $4 \pm 0$

10.5.2. T:  $9.28125 \pm 0.281125$ ; S:  $9 \pm 0$

10.5.3. T:  $60.75 \pm 1$ ; S:  $60 \pm 0$

10.5.4. T:  $1.1167 \pm 0.0833$ ; S:  $1.1000 \pm 0.0167$

10.5.5. T:  $0.3235 \pm 0.0026$ ; S:  $0.3217 \pm 0.000065$

10.5.6. T:  $0.6478 \pm 0.0052$ ; S:  $0.6438 \pm 0.000033$

10.5.7. T:  $2.8833 \pm 0.0834$ ; S:  $2.9000 \pm 0.0167$

10.5.8. T:  $1.1170 \pm 0.0077$ ; S:  $1.1114 \pm 0.0002$

10.5.9. T:  $1.097 \pm 0.0147$ ; S:  $1.089 \pm 0.0003$

10.5.10. T:  $3.63 \pm 0.087$ ; S:  $3.62 \pm 0.032$

10.6.1.  $\frac{(t+4)^4}{4} + C$

10.6.2.  $\frac{(t^2-9)^{5/2}}{5} + C$

10.6.3.  $\frac{(e^{t^2}+16)^2}{4} + C$

10.6.4.  $\cos t - \frac{2}{3} \cos^3 t + C$

10.6.5.  $\frac{\tan^2 t}{2} + C$

10.6.6.  $\ln|t^2+t+3| + C$

10.6.7.  $\frac{1}{8} \ln|1-4/t^2| + C$

10.6.8.  $\frac{1}{25} \tan(\arcsin(t/5)) + C = \frac{t}{25\sqrt{25-t^2}} + C$

10.6.9.  $\frac{2}{3} \sqrt{\sin 3t} + C$

10.6.10.  $t \tan t + \ln|\cos t| + C$

10.6.11.  $2\sqrt{e^t+1} + C$

10.6.12.  $\frac{3t}{8} + \frac{\sin 2t}{4} + \frac{\sin 4t}{32} + C$

10.6.13.  $\frac{\ln|t|}{3} - \frac{\ln|t+3|}{3} + C$

10.6.14.  $\frac{-1}{\sin \arctan t} + C = -\sqrt{1+t^2}/t + C$

10.6.15.  $\frac{-1}{2(1+\tan t)^2} + C$

10.6.16.  $\frac{(t^2+1)^{5/2}}{5} - \frac{(t^2+1)^{3/2}}{3} + C$

10.6.17.  $\frac{e^t \sin t - e^t \cos t}{2} + C$

10.6.18.  $\frac{(t^{3/2}+47)^4}{6} + C$

10.6.19.  $\frac{2}{3(2-t^2)^{3/2}} - \frac{1}{(2-t^2)^{1/2}} + C$

10.6.20.  $\frac{\ln|\sin(\arctan(2t/3))|}{9} + C = \frac{(\ln(4t^2) - \ln(9+4t^2))/18}{9} + C$

10.6.21.  $\frac{(\arctan(2t))^2}{4} + C$

10.6.22.  $\frac{3 \ln|t+3|}{4} + \frac{\ln|t-1|}{4} + C$

10.6.23.  $\frac{\cos^7 t}{7} - \frac{\cos^5 t}{5} + C$

10.6.24.  $\frac{-1}{t-3} + C$

- 10.6.25.  $\frac{-1}{\ln t} + C$
- 10.6.26.  $\frac{t^2(\ln t)^2}{2} - \frac{t^2 \ln t}{2} + \frac{t^2}{4} + C$
- 10.6.27.  $(t^3 - 3t^2 + 6t - 6)e^t + C$
- 10.6.28.  $\frac{5 + \sqrt{5}}{10} \ln(2t + 1 - \sqrt{5}) + \frac{5 - \sqrt{5}}{10} \ln(2t + 1 + \sqrt{5}) + C$
- 11.1.1.  $15/2$
- 11.1.2.  $5$
- 11.1.3.  $16/5$
- 11.1.5.  $\bar{x} = 45/28, \bar{y} = 93/70$
- 11.1.6.  $\bar{x} = 0, \bar{y} = 4/(3\pi)$
- 11.1.7.  $\bar{x} = 1/2, \bar{y} = 2/5$
- 11.1.8.  $\bar{x} = 0, \bar{y} = 8/5$
- 11.1.9.  $\bar{x} = 4/7, \bar{y} = 2/5$
- 11.1.10.  $\bar{x} = \bar{y} = 1/5$
- 11.1.11.  $\bar{x} = 0, \bar{y} = 28/(9\pi)$
- 11.1.12.  $\bar{x} = \bar{y} = 28/(9\pi)$
- 11.1.13.  $\bar{x} = 0, \bar{y} = 244/(27\pi) \approx 2.88$
- 11.2.1.  $\infty$
- 11.2.2.  $1/2$
- 11.2.3. diverges
- 11.2.4. diverges
- 11.2.5.  $1$
- 11.2.6. diverges
- 11.2.7.  $2$
- 11.2.8. diverges
- 11.2.9.  $\pi/6$
- 11.2.10. diverges,  $0$
- 11.2.11. diverges,  $0$
- 11.2.12. diverges, no CPV
- 11.2.13.  $\pi$
- 11.2.14. 80 mph: 90.8 to 95.3 N  
90 mph: 114.9 to 120.6 N  
100.9 mph: 144.5 to 151.6 N
- 11.3.2.  $\mu = 1/c, \sigma = 1/c$
- 11.3.3.  $\mu = (a + b)/2, \sigma = (a - b)^2/12$
- 11.3.4.  $7/2$
- 11.3.5.  $21/2$
- 11.3.9.  $r = 6$
- 11.4.1.  $(22\sqrt{22} - 8)/27$
- 11.4.2.  $\ln(2) + 3/8$
- 11.4.3.  $a + a^3/3$
- 11.4.4.  $\ln((\sqrt{2} + 1)/\sqrt{3})$
- 11.4.6.  $3/4$
- 11.4.7.  $\approx 3.82$
- 11.4.8.  $\approx 1.01$
- 11.4.9.  $\sqrt{1 + e^2} - \sqrt{2} + \frac{1}{2} \ln \left( \frac{\sqrt{1 + e^2} - 1}{\sqrt{1 + e^2} + 1} \right) + \frac{1}{2} \ln(3 + 2\sqrt{2})$
- 11.5.1.  $8\pi\sqrt{3} - \frac{16\pi\sqrt{2}}{3}$
- 11.5.3.  $\frac{730\pi\sqrt{730}}{27} - \frac{10\pi\sqrt{10}}{27}$
- 11.5.4.  $\pi + 2\pi e + \frac{1}{4}\pi e^2 - \frac{\pi}{4e^2} - \frac{2\pi}{e}$
- 11.5.6.  $8\pi^2$
- 11.5.7.  $2\pi + \frac{8\pi^2}{3\sqrt{3}}$
- 11.5.8.  $a > b: 2\pi b^2 + \frac{2\pi a^2 b}{\sqrt{a^2 - b^2}} \arcsin(\sqrt{a^2 - b^2}/a),$   
 $a < b: 2\pi b^2 + \frac{2\pi a^2 b}{\sqrt{b^2 - a^2}} \ln \left( \frac{b}{a} + \frac{\sqrt{b^2 - a^2}}{a} \right)$
- 12.1.2.  $\theta = \arctan(3)$

- 12.1.3.**  $r = -4 \csc \theta$   
**12.1.4.**  $r = \sec \theta \csc^2 \theta$   
**12.1.5.**  $r = \sqrt{5}$   
**12.1.6.**  $r^2 = \sin \theta \sec^3 \theta$   
**12.1.7.**  $r \sin \theta = \sin(r \cos \theta)$   
**12.1.8.**  $r = 2/(\sin \theta - 5 \cos \theta)$   
**12.1.9.**  $r = 2 \sec \theta$   
**12.1.10.**  $0 = r^2 \cos^2 \theta - r \sin \theta + 1$   
**12.1.11.**  $0 = 3r^2 \cos^2 \theta - 2r \cos \theta - r \sin \theta$   
**12.1.12.**  $r = \sin \theta$   
**12.1.21.**  $(x^2 + y^2)^2 = 4x^2y - (x^2 + y^2)y$   
**12.1.22.**  $(x^2 + y^2)^{3/2} = y^2$   
**12.1.23.**  $x^2 + y^2 = x^2y^2$   
**12.1.24.**  $x^4 + x^2y^2 = y^2$   
**12.2.1.**  $(\theta \cos \theta + \sin \theta)/(-\theta \sin \theta + \cos \theta),$   
 $(\theta^2 + 2)/(-\theta \sin \theta + \cos \theta)^3$   
**12.2.2.**  $\frac{\cos \theta + 2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta - \sin \theta},$   
 $\frac{3(1 + \sin \theta)}{(\cos^2 \theta - \sin^2 \theta - \sin \theta)^3}$   
**12.2.3.**  $(\sin^2 \theta - \cos^2 \theta)/(2 \sin \theta \cos \theta),$   
 $-1/(4 \sin^3 \theta \cos^3 \theta)$   
**12.2.4.**  $\frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta}, \frac{2}{(\cos^2 \theta - \sin^2 \theta)^3}$   
**12.2.5.** undefined  
**12.2.6.**  $\frac{2 \sin \theta - 3 \sin^3 \theta}{3 \cos^3 \theta - 2 \cos \theta},$   
 $\frac{3 \cos^4 \theta - 3 \cos^2 \theta + 2}{2 \cos^3 \theta (3 \cos^2 \theta - 2)^3}$   
**12.3.1.** 1  
**12.3.2.**  $9\pi/2$   
**12.3.3.**  $\sqrt{3}/3$   
**12.3.4.**  $\pi/12 + \sqrt{3}/16$   
**12.3.5.**  $\pi a^2/4$   
**12.3.6.**  $41\pi/2$   
**12.3.7.**  $2 - \pi/2$   
**12.3.8.**  $\pi/12$   
**12.3.9.**  $3\pi/16$   
**12.3.10.**  $\pi/4 - 3\sqrt{3}/8$   
**12.3.11.**  $\pi/2 + 3\sqrt{3}/8$   
**12.3.12.** 1  
**12.3.13.**  $3/2 - \pi/4$   
**12.3.14.**  $\pi/3 + \sqrt{3}/2$   
**12.3.15.**  $\pi/3 - \sqrt{3}/4$   
**12.3.16.**  $4\pi^3/3$   
**12.3.17.**  $\pi^2$   
**12.3.18.**  $5\pi/24 - \sqrt{3}/4$   
**12.3.19.**  $7\pi/12 - \sqrt{3}$   
**12.3.20.**  $4\pi - \sqrt{15}/2 - 7 \arccos(1/4)$   
**12.3.21.**  $3\pi^3$   
**12.4.6.**  $x = t - \frac{\sin(t)}{2}, t = 1 - \frac{\cos(t)}{2}$   
**12.4.7.**  $x = 4 \cos t - \cos(4t),$   
 $y = 4 \sin t - \sin(4t)$   
**12.4.8.**  $x = 2 \cos t + \cos(2t),$   
 $y = 2 \sin t - \sin(2t)$   
**12.4.9.**  $x = \cos t + t \sin t,$   
 $y = \sin t - t \cos t$   
**12.5.1.** There is a horizontal tangent at all multiples of  $\pi$ .  
**12.5.2.**  $9\pi/4$   
**12.5.3.**  $\int_0^{2\pi} \frac{1}{2} \sqrt{5 - 4 \cos t} dt$   
**12.5.4.** Four points:  
 $\left( \frac{-3 - 3\sqrt{5}}{4}, \pm \sqrt{\frac{5 - \sqrt{5}}{8}} \right),$   
 $\left( \frac{-3 + 3\sqrt{5}}{4}, \pm \sqrt{\frac{5 + \sqrt{5}}{8}} \right)$

- 12.5.5.**  $11\pi/3$   
**12.5.6.**  $32/3$   
**12.5.7.**  $2\pi$   
**12.5.8.**  $16/3$   
**12.5.9.**  $(\pi/2, 1)$   
**12.5.10.**  $5\pi^3/6$   
**12.5.11.**  $2\pi^2$   
**12.5.12.**  $(2\pi\sqrt{4\pi^2 + 1} + \ln(2\pi + \sqrt{4\pi^2 + 1}))/2$   
**13.1.1.** 1  
**13.1.3.** 0  
**13.1.4.** 1  
**13.1.5.** 1  
**13.1.6.** 0  
**13.2.1.**  $\lim_{n \rightarrow \infty} n^2/(2n^2 + 1) = 1/2$   
**13.2.2.**  $\lim_{n \rightarrow \infty} 5/(2^{1/n} + 14) = 1/3$   
**13.2.3.**  $\sum_{n=1}^{\infty} \frac{1}{n}$  diverges, so  $\sum_{n=1}^{\infty} 3\frac{1}{n}$  diverges  
**13.2.4.**  $-3/2$   
**13.2.5.** 11  
**13.2.6.** 20  
**13.2.7.**  $3/4$   
**13.2.8.**  $3/2$   
**13.2.9.**  $3/10$   
**13.3.1.** diverges  
**13.3.2.** diverges  
**13.3.3.** converges  
**13.3.4.** converges  
**13.3.5.** converges  
**13.3.6.** converges  
**13.3.7.** diverges  
**13.3.8.** converges  
**13.3.9.**  $N = 5$   
**13.3.10.**  $N = 10$   
**13.3.11.**  $N = 1687$   
**13.3.12.** any integer greater than  $e^{200}$   
**13.4.1.** converges  
**13.4.2.** converges  
**13.4.3.** diverges  
**13.4.4.** converges  
**13.4.5.** 0.90  
**13.4.6.** 0.95  
**13.5.1.** converges  
**13.5.2.** converges  
**13.5.3.** converges  
**13.5.4.** diverges  
**13.5.5.** diverges  
**13.5.6.** diverges  
**13.5.7.** converges  
**13.5.8.** diverges  
**13.5.9.** converges  
**13.5.10.** diverges  
**13.6.1.** converges absolutely  
**13.6.2.** diverges  
**13.6.3.** converges conditionally  
**13.6.4.** converges absolutely  
**13.6.5.** converges conditionally  
**13.6.6.** converges absolutely  
**13.6.7.** diverges  
**13.6.8.** converges conditionally  
**13.7.5.** converges  
**13.7.6.** converges

- 13.7.7.** converges  
**13.7.8.** diverges  
**13.8.1.**  $R = 1, I = (-1, 1)$   
**13.8.2.**  $R = \infty, I = (-\infty, \infty)$   
**13.8.3.**  $R = e, I = (-e, e)$   
**13.8.4.**  $R = e, I = (2 - e, 2 + e)$   
**13.8.5.**  $R = 0$ , converges only when  $x = 2$   
**13.8.6.**  $R = 1, I = [-6, -4]$   
**13.9.1.** the alternating harmonic series  
**13.9.2.**  $\sum_{n=0}^{\infty} (n+1)x^n$   
**13.9.3.**  $\sum_{n=0}^{\infty} (n+1)(n+2)x^n$   
**13.9.4.**  $\sum_{n=0}^{\infty} \frac{(n+1)(n+2)}{2} x^n, R = 1$   
**13.9.5.**  $C + \sum_{n=0}^{\infty} \frac{-1}{(n+1)(n+2)} x^{n+2}$   
**13.10.1.**  $\sum_{n=0}^{\infty} (-1)^n x^{2n} / (2n)!, R = \infty$   
**13.10.2.**  $\sum_{n=0}^{\infty} x^n / n!, R = \infty$   
**13.10.3.**  $\sum_{n=0}^{\infty} (-1)^n \frac{(x-5)^n}{5^{n+1}}, R = 5$   
**13.10.4.**  $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-1)^n}{n}, R = 1$   
**13.10.5.**  $\ln(2) + \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-2)^n}{n2^n}, R = 2$   
**13.10.6.**  $\sum_{n=0}^{\infty} (-1)^n (n+1)(x-1)^n, R = 1$   
**13.10.7.**  $1 + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{n!2^n} x^n =$   
 $1 + \sum_{n=1}^{\infty} \frac{(2n-1)!}{2^{2n-1}(n-1)!n!} x^n, R = 1$   
**13.10.8.**  $x + x^3/3$   
**13.10.9.**  $\sum_{n=0}^{\infty} (-1)^n x^{4n+1} / (2n)!$   
**13.10.10.**  $\sum_{n=0}^{\infty} (-1)^n x^{n+1} / n!$   
**13.11.1.**  $1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \cdots + \frac{x^{12}}{12!}$   
**13.11.2.** 1000; 8  
**13.11.3.**  $x + \frac{x^3}{3} + \frac{2x^5}{15}$ , error  $\pm 1.27$ .  
**13.12.1.** diverges  
**13.12.2.** converges  
**13.12.3.** converges  
**13.12.4.** diverges  
**13.12.5.** diverges  
**13.12.6.** diverges  
**13.12.7.** converges  
**13.12.8.** converges  
**13.12.9.** converges  
**13.12.10.** converges  
**13.12.11.** converges  
**13.12.12.** converges  
**13.12.13.** converges  
**13.12.14.** converges  
**13.12.15.** converges  
**13.12.16.** converges  
**13.12.17.** diverges  
**13.12.18.**  $(-\infty, \infty)$   
**13.12.19.**  $(-3, 3)$   
**13.12.20.**  $(-3, 3)$   
**13.12.21.**  $(-1, 1)$

- 13.12.22. radius is 0—it converges only when  $x = 0$
- 13.12.23.  $(-\sqrt{3}, \sqrt{3})$
- 13.12.24.  $(-\infty, \infty)$
- 13.12.25.  $\sum_{n=0}^{\infty} \frac{(\ln(2))^n}{n!} x^n$
- 13.12.26.  $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} x^{n+1}$
- 13.12.27.  $\sum_{n=0}^{\infty} \frac{2}{2n+1} x^{2n+1}$
- 13.12.28.  $1 + x/2 + \sum_{n=2}^{\infty} (-1)^{n+1} \frac{1 \cdot 3 \cdot 5 \cdots (2n-3)}{2^n n!} x^n$
- 13.12.29.  $\sum_{n=0}^{\infty} (-1)^n x^{2n}$
- 13.12.30.  $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$
- 13.12.31.  $\pi = \sum_{n=0}^{\infty} (-1)^n \frac{4}{2n+1}$
- 14.1.6.  $3, \sqrt{26}, \sqrt{29}$ .
- 14.1.7.  $\sqrt{14}, 2\sqrt{14}, 3\sqrt{14}$ .
- 14.1.8.  $(x-1)^2 + (y-1)^2 + (z-1)^2 = 4$ .
- 14.1.9.  $(x-2)^2 + (y+1)^2 + (z-3)^2 = 25$ .
- 14.1.11.  $(x-2)^2 + (y-1)^2 + (z+1)^2 = 16$ ,  
 $(y-1)^2 + (z+1)^2 = 12$
- 14.2.6.  $\sqrt{10}, \langle 0, -2 \rangle, \langle 2, 8 \rangle, 2, 2\sqrt{17},$   
 $\langle -2, -6 \rangle$
- 14.2.7.  $\sqrt{14}, \langle 0, 4, 0 \rangle, \langle 2, 0, 6 \rangle, 4, 2\sqrt{10},$   
 $\langle -2, -4, -6 \rangle$
- 14.2.8.  $\sqrt{2}, \langle 0, -2, 3 \rangle, \langle 2, 2, -1 \rangle, \sqrt{13}, 3,$   
 $\langle -2, 0, -2 \rangle$
- 14.2.9.  $\sqrt{3}, \langle 1, -1, 4 \rangle, \langle 1, -1, -2 \rangle, 3\sqrt{2}, \sqrt{6},$   
 $\langle -2, 2, -2 \rangle$
- 14.2.10.  $\sqrt{14}, \langle 2, 1, 0 \rangle, \langle 4, 3, 2 \rangle, \sqrt{5}, \sqrt{29},$   
 $\langle -6, -4, -2 \rangle$
- 14.2.11.  $\langle -3, -3, -11 \rangle,$   
 $\langle -3/\sqrt{139}, -3/\sqrt{139}, -11/\sqrt{139} \rangle$   
 $\langle -12/\sqrt{139}, -12/\sqrt{139}, -44/\sqrt{139} \rangle$
- 14.2.12.  $\langle 0, 0, 0 \rangle$
- 14.2.13.  $0; \langle -r\sqrt{3}/2, r/2 \rangle; \langle 0, -12r \rangle;$  where  
 $r$  is the radius of the clock
- 14.3.1. 3
- 14.3.2. 0
- 14.3.3. 2
- 14.3.4. -6
- 14.3.5. 42
- 14.3.6.  $\sqrt{6}/\sqrt{7}, \approx 0.39$
- 14.3.7.  $-11\sqrt{14}\sqrt{29}/406, \approx 2.15$
- 14.3.8.  $0, \pi/2$
- 14.3.9.  $1/2, \pi/3$
- 14.3.10.  $-1/\sqrt{3}, \approx 2.19$
- 14.3.11.  $\arccos(1/\sqrt{3}) \approx 0.96$
- 14.3.12.  $\sqrt{5}, \langle 1, 2, 0 \rangle.$
- 14.3.13.  $3\sqrt{14}/7, \langle 9/7, 6/7, 3/7 \rangle.$
- 14.3.14.  $\langle 0, 5 \rangle, \langle 5\sqrt{3}, 0 \rangle$
- 14.3.15.  $\langle 0, 15\sqrt{2}/2 \rangle, \langle 15\sqrt{2}/2, 0 \rangle$
- 14.3.16. Any vector of the form  
 $\langle a, -7a/2, -2a \rangle$
- 14.3.17.  $\langle 1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3} \rangle$
- 14.3.18. No.
- 14.3.19. Yes.
- 14.4.1.  $\langle 1, -2, 1 \rangle$
- 14.4.2.  $\langle 4, -6, -2 \rangle$
- 14.4.3.  $\langle -7, 13, -9 \rangle$



- 14.4.4.  $\langle 0, -1, 0 \rangle$   
 14.4.5. 3  
 14.4.6.  $21/\sqrt{2}$   
 14.4.7. 1  
 14.5.1.  $(x - 6) + (y - 2) + (z - 1) = 0$   
 14.5.2.  $4(x + 1) + 5(y - 2) - (z + 3) = 0$   
 14.5.3.  $(x - 1) - (y - 2) = 0$   
 14.5.4.  $-2(x - 1) + 3y - 2z = 0$   
 14.5.5.  $4(x - 1) - 6y = 0$   
 14.5.6.  $x + 3y = 0$   
 14.5.7.  $\langle 1, 0, 3 \rangle + t\langle 0, 2, 1 \rangle$   
 14.5.8.  $\langle 1, 0, 3 \rangle + t\langle 1, 2, -1 \rangle$   
 14.5.9.  $t\langle 1, 1, -1 \rangle$   
 14.5.10.  $-2/5, 13/5$   
 14.5.12. neither  
 14.5.13. parallel  
 14.5.14. intersect  
 14.5.15. same line  
 14.5.19.  $7/\sqrt{3}$   
 14.5.20.  $4/\sqrt{14}$   
 14.5.21.  $\sqrt{131}/\sqrt{14}$   
 14.5.22.  $\sqrt{68}/3$   
 14.5.23.  $\sqrt{42}/7$   
 14.5.24.  $\sqrt{21}/6$   
 14.6.1.  $(\sqrt{2}, \pi/4, 1),$   
 $(\sqrt{3}, \pi/4, \arccos(1/\sqrt{3}));$   
 $(7\sqrt{2}, 7\pi/4, 5),$   
 $(\sqrt{123}, 7\pi/4, \arccos(5/\sqrt{123}));$   
 $(1, 1, 1), (\sqrt{2}, 1, \pi/4); (0, 0, -\pi),$   
 $(\pi, 0, \pi)$   
 14.6.2.  $r^2 + z^2 = 4$   
 14.6.3.  $r \cos \theta = 0$   
 14.6.4.  $r^2 + 2z^2 + 2z - 5 = 0$   
 14.6.5.  $z = e^{-r^2}$   
 14.6.6.  $z = r$   
 14.6.7.  $\sin \theta = 0$   
 14.6.8.  $1 = \rho \cos \phi$   
 14.6.9.  $\rho = 2 \sin \theta \sin \phi.$   
 14.6.10.  $\rho \sin \phi = 2$   
 14.6.11.  $\cos \phi = 1/\sqrt{2}$   
 14.6.13.  $z = mr; \cot \phi = m$  if  $m \neq 0, \phi = 0$   
 if  $m = 0$   
 14.6.14. A sphere with radius  $1/2$ , center at  
 $(0, 1/2, 0)$   
 14.6.15.  $0 < \theta < \pi/2, 0 < \phi < \pi/2, \rho > 0;$   
 $0 < \theta < \pi/2, r > 0, z > 0$   
 15.1.5.  $\langle 3 \cos t, 3 \sin t, 2 - 3 \sin t \rangle$   
 15.1.6.  $\langle 0, t \cos t, t \sin t \rangle$   
 15.2.1.  $\langle 2t, 0, 1 \rangle, \mathbf{r}'/\sqrt{1 + 4t^2}$   
 15.2.2.  $\langle -\sin t, 2 \cos 2t, 2t \rangle,$   
 $\mathbf{r}'/\sqrt{\sin^2 t + 4 \cos^2(2t) + 4t^2}$   
 15.2.3.  $\langle -e^t \sin(e^t), e^t \cos(e^t), \cos t \rangle,$   
 $\mathbf{r}'/\sqrt{e^{2t} + \cos^2 t}$   
 15.2.4.  $\langle \sqrt{2}/2, \sqrt{2}/2, \pi/4 \rangle +$   
 $t\langle -\sqrt{2}/2, \sqrt{2}/2, 1 \rangle$   
 15.2.5.  $\langle 1/2, \sqrt{3}/2, -1/2 \rangle +$   
 $t\langle -\sqrt{3}/2, 1/2, 2\sqrt{3} \rangle$   
 15.2.6.  $2/\sqrt{5}/\sqrt{4 + \pi^2}$   
 15.2.7.  $7\sqrt{5}\sqrt{17}/85, -9\sqrt{5}\sqrt{17}/85$   
 15.2.9.  $\langle 0, t \cos t, t \sin t \rangle, \langle 0, \cos t -$   
 $t \sin t, \sin t + t \cos t \rangle, \mathbf{r}'/\sqrt{1 + t^2},$   
 $\sqrt{1 + t^2}$   
 15.2.10.  $\langle \sin t, 1 - \cos t, t^2/2 \rangle$   
 15.2.11.  $t = 4$   
 15.2.12. 37, 1

- 15.2.13.  $\langle t^2/2, t^3/3, \sin t \rangle$
- 15.2.16.  $(1, 1, 1)$  when  $t = 1$  and  $s = 0$ ;  
 $\theta = \arccos(3/\sqrt{14})$ ; no
- 15.2.17.  $-6x + (y - \pi) = 0$
- 15.2.18.  $-x/\sqrt{2} + y/\sqrt{2} + 6z = 0$
- 15.2.19.  $(-1, -3, 1)$
- 15.2.20.  $\langle 1/\sqrt{2}, 1/\sqrt{2}, 0 \rangle + t\langle -1, 1, 6\sqrt{2} \rangle$
- 15.3.1.  $2\pi\sqrt{13}$
- 15.3.2.  $(-8 + 13\sqrt{13})/27$
- 15.3.3.  $\sqrt{5}/2 + \ln(\sqrt{5} + 2)/4$
- 15.3.4.  $(85\sqrt{85} - 13\sqrt{13})/27$
- 15.3.5.  $\int_0^5 \sqrt{1 + e^{2t}} dt$
- 15.3.6.  $2\sqrt{2}/(2 + 4t^2)^{3/2}$
- 15.3.7.  $2\sqrt{2}/(1 + 8t^2)^{3/2}$
- 15.3.8.  $2\sqrt{1 + 9t^2 + 9t^4}/(1 + 4t^2 + 9t^4)^{3/2}$
- 15.3.9.  $12\sqrt{17}/289$
- 15.4.1.  $\langle -\sin t, \cos t, 1 \rangle, \langle -\cos t, -\sin t, 0 \rangle,$   
 $0, 1$
- 15.4.2.  $\langle -\sin t, \cos t, 2t \rangle, \langle -\cos t, -\sin t, 2 \rangle,$   
 $4t/\sqrt{4t^2 + 1}, \sqrt{4t^2 + 5}/\sqrt{4t^2 + 1}$
- 15.4.3.  $\langle -\sin t, \cos t, e^t \rangle,$   
 $\langle -\cos t, -\sin t, e^t \rangle, e^{2t}/\sqrt{e^{2t} + 1},$   
 $\sqrt{2e^{2t} + 1}/\sqrt{e^{2t} + 1}$
- 15.4.4.  $\langle e^t, \cos t, e^t \rangle, \langle e^t, -\sin t, e^t \rangle,$   
 $(2e^{2t} - \cos t \sin t)/\sqrt{2e^{2t} + \cos^2 t},$   
 $\sqrt{2}e^t |\cos t + \sin t|/\sqrt{2e^{2t} + \cos^2 t}$
- 15.4.5.  $\langle -3 \sin t, 2 \cos t, 0 \rangle, \langle 3 \cos t, 2 \sin t, 0 \rangle$
- 15.4.6.  $\langle -3 \sin t, 2 \cos t + 0.1, 0 \rangle,$   
 $\langle 3 \cos t, 2 \sin t + t/10, 0 \rangle$
- 15.4.7.  $\langle -3 \sin t, 2 \cos t, 1 \rangle, \langle 3 \cos t, 2 \sin t, t \rangle$
- 15.4.8.  $\langle -3 \sin t, 2 \cos t + 1/10, 1 \rangle,$   
 $\langle 3 \cos t, 2 \sin t + t/10, t \rangle$
- 16.1.1.  $z = y^2, z = x^2, z = 0$ , lines of slope 1
- 16.1.2.  $z = |y|, z = |x|, z = 2|x|$ , diamonds
- 16.1.3.  $z = e^{-y^2} \sin(y^2), z = e^{-x^2} \sin(x^2),$   
 $z = e^{-2x^2} \sin(2x^2)$ , circles
- 16.1.4.  $z = -\sin(y), z = \sin(x), z = 0$ ,  
lines of slope 1
- 16.1.5.  $z = y^4, z = x^4, z = 0$ , hyperbolas
- 16.1.6. (a)  $\{(x, y) \mid |x| \leq 3 \text{ and } |y| \geq 2\}$   
(b)  $\{(x, y) \mid 1 \leq x^2 + y^2 \leq 3\}$   
(c)  $\{(x, y) \mid x^2 + 4y^2 \leq 16\}$
- 16.2.1. No limit; use  $x = 0$  and  $y = 0$ .
- 16.2.2. No limit; use  $x = 0$  and  $x = y$ .
- 16.2.3. No limit; use  $x = 0$  and  $x = y$ .
- 16.2.4. Limit is zero.
- 16.2.5. Limit is 1.
- 16.2.6. Limit is zero.
- 16.2.7. Limit is  $-1$ .
- 16.2.8. Limit is zero.
- 16.2.9. No limit; use  $x = 0$  and  $y = 0$ .
- 16.2.10. Limit is zero.
- 16.2.11. Limit is  $-1$ .
- 16.2.12. Limit is zero.
- 16.3.1.  $-2xy \sin(x^2y), -x^2 \sin(x^2y) + 3y^2$
- 16.3.2.  $(y^2 - x^2y)/(x^2 + y)^2, x^3/(x^2 + y)^2$
- 16.3.3.  $2xe^{x^2+y^2}, 2ye^{x^2+y^2}$
- 16.3.4.  $y \ln(xy) + y, x \ln(xy) + x$
- 16.3.5.  $-x/\sqrt{1 - x^2 - y^2},$   
 $-y/\sqrt{1 - x^2 - y^2}$
- 16.3.6.  $\tan y, x \sec^2 y$
- 16.3.7.  $-1/(x^2y), -1/(xy^2)$
- 16.3.8.  $z = -2(x - 1) - 3(y - 1) - 1$
- 16.3.9.  $z = 1$

$$16.3.10. z = 6(x - 3) + 3(y - 1) + 10$$

$$16.3.11. z = (x - 2) + 4(y - 1/2)$$

$$16.3.12. \mathbf{r}(t) = \langle 2, 1, 4 \rangle + t\langle 2, 4, -1 \rangle$$

$$16.4.1. 4xt \cos(x^2 + y^2) + 6yt^2 \cos(x^2 + y^2)$$

$$16.4.2. 2xy \cos t + 2x^2t$$

$$16.4.3. 2xyt \cos(st) + 2x^2s, 2xys \cos(st) + 2x^2t$$

$$16.4.4. 2xy^2t - 4yx^2s, 2xy^2s + 4yx^2t$$

$$16.4.5. x/z, 3y/(2z)$$

$$16.4.6. -2x/z, -y/z$$

$$16.4.7. (a) V' = (nR - 0.2V)/P$$

$$(b) P' = (nR + 0.6P)/2V$$

$$(c) T' = (3P - 0.4V)/(nR)$$

$$16.5.1. 9\sqrt{5}/5$$

$$16.5.2. \sqrt{2} \cos 3$$

$$16.5.3. e\sqrt{2}(\sqrt{3} - 1)/4$$

$$16.5.4. \sqrt{3} + 5$$

$$16.5.5. -\sqrt{6}(2 + \sqrt{3})/72$$

$$16.5.6. -1/5, 0$$

$$16.5.7. 4(x - 2) + 8(y - 1) = 0$$

$$16.5.8. 2(x - 3) + 3(y - 2) = 0$$

$$16.5.9. \langle -1, -1 - \cos 1, -\cos 1 \rangle, \\ -\sqrt{2 + 2 \cos 1 + 2 \cos^2 1}$$

16.5.10. Any direction perpendicular to  $\nabla T = \langle 1, 1, 1 \rangle$ , for example,  $\langle -1, 1, 0 \rangle$

$$16.5.11. 2(x - 1) - 6(y - 1) + 6(z - 3) = 0$$

$$16.5.12. 6(x - 1) + 3(y - 2) + 2(z - 3) = 0$$

$$16.5.13. \langle 2 + 4t, -3 - 12t, -1 - 8t \rangle$$

$$16.5.14. \langle 4 + 8t, 2 + 4t, -2 - 36t \rangle$$

$$16.5.15. \langle 4 + 8t, 2 + 20t, 6 - 12t \rangle$$

$$16.5.16. \langle 0, 1 \rangle, \langle 4/5, -3/5 \rangle$$

$$16.5.18. (a) \langle 4, 9 \rangle (b) \langle -81, 2 \rangle \text{ or } \langle 81, -2 \rangle$$

16.5.19. in the direction of  $\langle 8, 1 \rangle$

$$16.5.20. \nabla g(-1, 3) = \langle 2, 1 \rangle$$

$$16.6.1. f_{xx} = (2x^3y - 6xy^3)/(x^2 + y^2)^3, \\ f_{yy} = (2xy^3 - 6x^3y)/(x^2 + y^2)^3$$

$$16.6.2. f_x = 3x^2y^2, f_y = 2x^3y + 5y^4, \\ f_{xx} = 6xy^2, f_{yy} = 2x^3 + 20y^3, \\ f_{xy} = 6x^2y$$

$$16.6.3. f_x = 12x^2 + y^2, f_y = 2xy, \\ f_{xx} = 24x, f_{yy} = 2x, f_{xy} = 2y$$

$$16.6.4. f_x = \sin y, f_y = x \cos y, f_{xx} = 0, \\ f_{yy} = -x \sin y, f_{xy} = \cos y$$

$$16.6.5. f_x = 3 \cos(3x) \cos(2y), \\ f_y = -2 \sin(3x) \sin(2y), \\ f_{xy} = -6 \cos(3x) \sin(2y), \\ f_{yy} = -4 \sin(3x) \cos(2y), \\ f_{xx} = -9 \sin(3x) \cos(2y)$$

$$16.6.6. f_x = e^{x+y^2}, f_y = 2ye^{x+y^2}, \\ f_{xx} = e^{x+y^2}, \\ f_{yy} = 4y^2e^{x+y^2} + 2e^{x+y^2}, \\ f_{xy} = 2ye^{x+y^2}$$

$$16.6.7. f_x = \frac{3x^2}{2(x^3 + y^4)}, f_y = \frac{2y^3}{x^3 + y^4}, \\ f_{xx} = \frac{3x}{x^3 + y^4} - \frac{9x^4}{2(x^3 + y^4)^2}, \\ f_{yy} = \frac{6y^2}{x^3 + y^4} - \frac{8y^6}{(x^3 + y^4)^2}, \\ f_{xy} = \frac{-6x^2y^3}{(x^3 + y^4)^2}$$

$$16.6.8. z_x = \frac{-x}{16z}, z_y = \frac{-y}{4z}, \\ z_{xx} = -\frac{16z^2 + x^2}{16^2z^3}, \\ z_{yy} = -\frac{4z^2 + y^2}{16z^3}, \\ z_{xy} = \frac{-xy}{64z^3}$$

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- 16.6.9.  $z_x = -\frac{y+z}{x+y}$ ,  $z_y = -\frac{x+z}{x+y}$ ,  
 $z_{xx} = 2\frac{y+z}{(x+y)^2}$ ,  $z_{yy} = 2\frac{x+z}{(x+y)^2}$ ,  
 $z_{xy} = \frac{2z}{(x+y)^2}$
- 16.7.1. minimum at (1, -1)  
 16.7.2. none  
 16.7.3. none  
 16.7.4. maximum at (1, -1/6)  
 16.7.5. none  
 16.7.6. minimum at (2, -1)  
 16.7.7.  $f(2, 2) = -2$ ,  $f(2, 0) = 4$   
 16.7.8. a cube  $1/\sqrt[3]{2}$  on a side  
 16.7.9.  $65/3 \times 65/3 \times 130/3$   
 16.7.10. It has a square base, and is one and one half times as tall as wide. If the volume is  $V$  the dimensions are  $\sqrt[3]{2V/3} \times \sqrt[3]{2V/3} \times \sqrt[3]{9V/4}$ .  
 16.7.11.  $\sqrt{100/3}$   
 16.7.12.  $|ax_0 + by_0 + cz_0 - d|/\sqrt{a^2 + b^2 + c^2}$   
 16.7.13. The sides and bottom should all be 2/3 meter, and the sides should be bent up at angle  $\pi/3$ .  
 16.7.14. (3, 4/3)  
 16.7.16.  $|b|$  if  $b \leq 1/2$ , otherwise  $\sqrt{b - 1/4}$   
 16.7.17.  $|b|$  if  $b \leq 1/2$ , otherwise  $\sqrt{b - 1/4}$   
 16.7.19.  $1024/\sqrt{3}$   
 16.8.1. a cube  
 16.8.2.  $65/3 \times 65/3 \times 130/3$   
 16.8.3. It has a square base, and is one and one half times as tall as wide. If the volume is  $V$  the dimensions are  $\sqrt[3]{2V/3} \times \sqrt[3]{2V/3} \times \sqrt[3]{9V/4}$ .  
 16.8.4.  $|ax_0 + by_0 + cz_0 - d|/\sqrt{a^2 + b^2 + c^2}$   
 16.8.5. (0, 0, 1), (0, 0, -1)  
 16.8.6.  $\sqrt[3]{4V} \times \sqrt[3]{4V} \times \sqrt[3]{V/16}$   
 16.8.7. Farthest:  $(-\sqrt{2}, \sqrt{2}, 2 + 2\sqrt{2})$ ;  
 closest: (2, 0, 0), (0, -2, 0)  
 16.8.8.  $x = y = z = 16$   
 16.8.9. (1, 2, 2)  
 16.8.10.  $(0, \sqrt{5}, -\sqrt{5})$ ,  $(0, -\sqrt{5}, \sqrt{5})$   
 16.8.11. standard \$65, deluxe \$75  
 16.8.12.  $x = 9$ ,  $\phi = \pi/3$   
 16.8.13. 35, -35  
 16.8.14. maximum  $e^4$ , no minimum  
 16.8.15. 5, -9/2  
 16.8.16. 3, 3, 3  
 16.8.17. a cube of side length  $2/\sqrt{3}$   
 17.1.1. 16  
 17.1.2. 4  
 17.1.3. 15/8  
 17.1.4. 1/2  
 17.1.5. 5/6  
 17.1.6.  $12 - 65/(2e)$ .  
 17.1.7. 1/2  
 17.1.8.  $\pi/64$   
 17.1.9.  $(2/9)2^{3/2} - (2/9)$   
 17.1.10.  $(1 - \cos(1))/4$   
 17.1.11.  $(2\sqrt{2} - 1)/6$   
 17.1.12.  $\pi - 2$   
 17.1.13.  $(e^9 - 1)/6$   
 17.1.14.  $\frac{4}{15} - \frac{\pi}{4}$   
 17.1.15. 1/3  
 17.1.16. 448

17.1.17.  $4/5$

17.1.18.  $8\pi$

17.1.19.  $2$

17.1.20.  $5/3$

17.1.21.  $81/2$

17.1.22.  $2a^3/3$

17.1.23.  $4\pi$

17.1.24.  $\pi/32$

17.1.25.  $31/8$

17.1.26.  $128/15$

17.1.27.  $1800\pi \text{ m}^3$

17.1.28. 
$$\frac{(e^2 + 8e + 16)}{15} \sqrt{e + 4} - \frac{5\sqrt{5}}{3} - \frac{e^{5/2}}{15} + \frac{1}{15}$$

17.1.30.  $16 - 8\sqrt{2}$

17.2.1.  $4\pi$

17.2.2.  $32\pi/3 - 4\sqrt{3}\pi$

17.2.3.  $(2 - \sqrt{2})\pi/3$

17.2.4.  $4/9$

17.2.5.  $5\pi/3$

17.2.6.  $\pi/6$

17.2.7.  $\pi/2$

17.2.8.  $\pi/2 - 1$

17.2.9.  $\sqrt{3}/4 + \pi/6$

17.2.10.  $8 + \pi$

17.2.11.  $\pi/12$

17.2.12.  $(1 - \cos(9))\pi/2$

17.2.13.  $-a^5/15$

17.2.14.  $12\pi$

17.2.15.  $\pi$

17.2.16.  $16/3$

17.2.17.  $21\pi$

17.2.19.  $2\pi$

17.3.1.  $\bar{x} = \bar{y} = 2/3$

17.3.2.  $\bar{x} = 4/5, \bar{y} = 8/15$

17.3.3.  $\bar{x} = 0, \bar{y} = 3\pi/16$

17.3.4.  $\bar{x} = 0, \bar{y} = 16/(15\pi)$

17.3.5.  $\bar{x} = 3/2, \bar{y} = 9/4$

17.3.6.  $\bar{x} = 6/5, \bar{y} = 12/5$

17.3.7.  $\bar{x} = 14/27, \bar{y} = 28/55$

17.3.8.  $(3/4, 2/5)$

17.3.9.  $\left(\frac{81\sqrt{3}}{80\pi}, 0\right)$

17.3.10.  $\bar{x} = \pi/2, \bar{y} = \pi/8$

17.3.11. 
$$M = \int_0^{2\pi} \int_0^{1+\cos\theta} (2 + \cos\theta)r \, dr \, d\theta,$$

$$M_x = \int_0^{2\pi} \int_0^{1+\cos\theta} \sin\theta(2 + \cos\theta)r^2 \, dr \, d\theta,$$

$$M_y = \int_0^{2\pi} \int_0^{1+\cos\theta} \cos\theta(2 + \cos\theta)r^2 \, dr \, d\theta.$$

17.3.12. 
$$M = \int_{-\pi/2}^{\pi/2} \int_0^{\cos\theta} (r + 1)r \, dr \, d\theta,$$

$$M_x = \int_{-\pi/2}^{\pi/2} \int_0^{\cos\theta} \sin\theta(r + 1)r^2 \, dr \, d\theta,$$

$$M_y = \int_{-\pi/2}^{\pi/2} \int_0^{\cos\theta} \cos\theta(r + 1)r^2 \, dr \, d\theta.$$

17.3.13. 
$$M = \int_{-\pi/2}^{\pi/2} \int_{\cos\theta}^{1+\cos\theta} r \, dr \, d\theta + \int_{\pi/2}^{3\pi/2} \int_0^{1+\cos\theta} r \, dr \, d\theta,$$

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$$M_x = \int_{-\pi/2}^{\pi/2} \int_{\cos \theta}^{1+\cos \theta} r^2 \sin \theta \, dr \, d\theta +$$

$$\int_{\pi/2}^{3\pi/2} \int_0^{1+\cos \theta} r^2 \sin \theta \, dr \, d\theta,$$

$$M_y = \int_{-\pi/2}^{\pi/2} \int_{\cos \theta}^{1+\cos \theta} r^2 \cos \theta \, dr \, d\theta +$$

$$\int_{\pi/2}^{3\pi/2} \int_0^{1+\cos \theta} r^2 \cos \theta \, dr \, d\theta.$$

17.6.3.  $5\pi/4$

17.6.4. 0

17.6.5.  $5\pi/4$

17.6.6.  $4/5$

17.6.7.  $256\pi/15$

17.6.8.  $4\pi^2$

17.6.9.  $\frac{3\pi}{16}$

17.4.1.  $\pi a \sqrt{h^2 + a^2}$

17.4.2.  $\pi a^2 \sqrt{m^2 + 1}$

17.4.3.  $\sqrt{3}/2$

17.4.4.  $\pi\sqrt{2}$

17.4.5.  $\pi\sqrt{2}/8$

17.4.6.  $\pi/2 - 1$

17.4.7.  $\frac{d^2 \sqrt{a^2 + b^2 + c^2}}{2abc}$

17.4.8.  $8\sqrt{3}\pi/3$

17.5.1.  $11/24$

17.5.2.  $623/60$

17.5.3.  $-3e^2/4 + 2e - 3/4$

17.5.4.  $1/20$

17.5.5.  $\pi/48$

17.5.6.  $11/84$

17.5.7.  $151/60$

17.5.8.  $\pi$

17.5.10.  $\frac{3\pi}{16}$

17.5.11. 32

17.5.12.  $64/3$

17.5.13.  $\bar{x} = \bar{y} = 0, \bar{z} = 16/15$

17.5.14.  $\bar{x} = \bar{y} = 0, \bar{z} = 1/3$

17.6.1.  $\pi/12$

17.6.2.  $\pi(1 - \sqrt{2}/2)$

17.6.10.  $\pi k h^2 a^2 / 12$

17.6.11.  $\pi k h a^3 / 6$

17.6.12.  $\pi^2 / 4$

17.6.13.  $4\pi / 5$

17.6.14.  $15\pi$

17.6.15.  $9k\pi(5\sqrt{2} - 2\sqrt{5})/20$

17.7.1.  $4\pi\sqrt{3}/3$

17.7.2. 0

17.7.3.  $2/3$

17.7.4.  $\frac{e^2 - 1}{2e^2}$

17.7.5. 36

17.7.6.  $32(\sqrt{2} + \ln(1 + \sqrt{2}))/3$

17.7.7.  $3 \cos(1) - 3 \cos(4)$

17.7.8.  $\pi(1 - \cos(1))/24$

17.7.10.  $(4/3)\pi abc$

18.2.1.  $13\sqrt{11}/4$

18.2.2. 0

18.2.3.  $3 \sin(4)/2$

18.2.4. 0

18.2.5.  $2e^3$

18.2.6. 128

18.2.7.  $(9e - 3)/2$

18.2.8.  $e^{e+1} - e^e - e^{1/e-1} + e^{1/e} + e^4/4 - e^{-4}/4$

- 18.2.9.  $1 + \sin(1) - \cos(1)$   
 18.2.10.  $3 \ln 3 - 2 \ln 2$   
 18.2.11.  $3/20 + 10 \ln(2)/7$   
 18.2.12.  $2 \ln 5 - 2 \ln 2 + 15/32$   
 18.2.13. 1  
 18.2.14. 0  
 18.2.15.  $21 + \cos(1) - \cos(8)$   
 18.2.16.  $(\ln 29 - \ln 2)/2$   
 18.2.17.  $2 \ln 2 + \pi/4 - 2$   
 18.2.18.  $1243/3$   
 18.2.19.  $\ln 2 + 11/3$   
 18.2.20.  $3 \cos(1) - \cos(2) - \cos(4) - \cos(8)$   
 18.2.21.  $-10/3$   
 18.3.1. no  $f$   
 18.3.2.  $x^4/4 - y^5/5$   
 18.3.3. no  $f$   
 18.3.4. no  $f$   
 18.3.5.  $y \sin x$   
 18.3.6. no  $f$   
 18.3.7.  $xyz$   
 18.3.8. 414  
 18.3.9. 6  
 18.3.10.  $1/e - \sin 3$   
 18.3.11.  $1/\sqrt{77} - 1/\sqrt{3}$   
 18.4.1. 1  
 18.4.2. 0  
 18.4.3.  $1/(2e) - 1/(2e^7) + e/2 - e^7/2$   
 18.4.4.  $1/2$   
 18.4.5.  $-1/6$   
 18.4.6.  $(2\sqrt{3} - 10\sqrt{5} + 8\sqrt{6})/3 - 2\sqrt{2}/5 + 1/5$   
 18.4.7.  $11/2 - \ln(2)$   
 18.4.8.  $2 - \pi/2$   
 18.4.9.  $-17/12$   
 18.4.10. 0  
 18.4.11.  $-\pi/2$   
 18.4.12.  $12\pi$   
 18.4.13.  $2 \cos(1) - 2 \sin(1) - 1$   
 18.5.1.  $-1, 0$   
 18.5.2.  $0, a + b$   
 18.5.3.  $(2b - a)/3, 0$   
 18.5.4.  $0, 1$   
 18.5.5.  $-2\pi, 0$   
 18.5.6.  $0, 2\pi$   
 18.6.3.  $25\sqrt{21}/4$   
 18.6.4.  $\pi\sqrt{21}$   
 18.6.5.  $\pi(5\sqrt{5} - 1)/6$   
 18.6.6.  $4\pi\sqrt{2}$   
 18.6.7.  $\pi a^2/2$   
 18.6.8.  $2\pi a(a - \sqrt{a^2 - b^2})$   
 18.6.9.  $\pi((1 + 4a^2)^{3/2} - 1)/6$   
 18.6.10.  $2\pi((1 + a^2)^{3/2} - 1)/3$   
 18.6.11.  $\pi a^2 - 2a^2$   
 18.6.12.  $\pi a^2 \sqrt{1 + k^2}/4$   
 18.6.13.  $A\sqrt{1 + a^2 + b^2}$   
 18.6.14.  $A\sqrt{k^2 + 1}$   
 18.6.15.  $8a^2$   
 18.7.1.  $(0, 0, 3/8)$   
 18.7.2.  $(11/20, 11/20, 3/10)$   
 18.7.3.  $(0, 0, 1364/425)$   
 18.7.4. on center axis,  $h/3$  above the base  
 18.7.5. 16  
 18.7.6. 7  
 18.7.7.  $-\pi$

- 18.7.8.  $-137/120$
- 18.7.9.  $-2/e$
- 18.7.10.  $\pi b^2(-4b^4 - 3b^2 + 6a^2b^2 + 6a^2)/6$
- 18.7.11.  $4/3 \text{ kg/s}$
- 18.7.12.  $24\epsilon_0$
- 18.8.1.  $-3\pi$
- 18.8.2.  $0$
- 18.8.3.  $-4\pi$
- 18.8.4.  $3\pi$
- 18.8.5.  $A(p(c-b) + q(a-c) + a-b)$
- 18.9.1. both are  $-45\pi/4$
- 18.9.2.  $a^2bc + ab^2c + abc^2$
- 18.9.3.  $e^2 - 2e + 7/2$
- 18.9.4.  $3$
- 18.9.5.  $384\pi/5$
- 18.9.6.  $\pi/3$
- 18.9.7.  $10\pi$
- 18.9.8.  $\pi/2$
- 19.1.2.  $y = \arctan t + C$
- 19.1.3.  $y = \frac{t^{n+1}}{n+1} + 1$
- 19.1.4.  $y = t \ln t - t + C$
- 19.1.5.  $y = n\pi$ , for any integer  $n$ .
- 19.1.6. none
- 19.1.7.  $y = \pm\sqrt{t^2 + C}$
- 19.1.8.  $y = \pm 1$ ,  $y = (1 + Ae^{2t})/(1 - Ae^{2t})$
- 19.1.9.  $y^4/4 - 5y = t^2/2 + C$
- 19.1.10.  $y = (2t/3)^{3/2}$
- 19.1.11.  $y = M + Ae^{-kt}$
- 19.1.12.  $\frac{10 \ln(15/2)}{\ln 5} \approx 2.52 \text{ minutes}$
- 19.1.13.  $y = \frac{M}{1 + Ae^{-Mkt}}$
- 19.1.14.  $y = 2e^{3t/2}$
- 19.1.15.  $t = -\frac{\ln 2}{k}$
- 19.1.16.  $600e^{-6 \ln 2/5} \approx 261 \text{ mg}$ ;  $\frac{5 \ln 300}{\ln 2} \approx 41 \text{ days}$
- 19.1.17.  $100e^{-200 \ln 2/191} \approx 48 \text{ mg}$ ;  
 $\frac{5730 \ln 50}{\ln 2} \approx 32339 \text{ years}$
- 19.1.18.  $y = y_0 e^{t \ln 2}$
- 19.1.19.  $500e^{-5 \ln 2/4} \approx 210 \text{ g}$
- 19.2.1.  $y = Ae^{-5t}$
- 19.2.2.  $y = Ae^{2t}$
- 19.2.3.  $y = Ae^{-\arctan t}$
- 19.2.4.  $y = Ae^{-t^3/3}$
- 19.2.5.  $y = 4e^{-t}$
- 19.2.6.  $y = -2e^{3t-3}$
- 19.2.7.  $y = e^{1+\cos t}$
- 19.2.8.  $y = e^2 e^{-e^t}$
- 19.2.9.  $y = 0$
- 19.2.10.  $y = 0$
- 19.2.11.  $y = 4t^2$
- 19.2.12.  $y = -2e^{(1/t)-1}$
- 19.2.13.  $y = e^{1-t^{-2}}$
- 19.2.14.  $y = 0$
- 19.2.15.  $k = \ln 5$ ,  $y = 100e^{-t \ln 5}$
- 19.2.16.  $k = -12/13$ ,  $y = \exp(-13t^{1/13})$
- 19.2.17.  $y = 10^6 e^{t \ln(3/2)}$
- 19.2.18.  $y = 10e^{-t \ln(2)/6}$
- 19.3.1.  $y = Ae^{-4t} + 2$
- 19.3.2.  $y = Ae^{2t} - 3$
- 19.3.3.  $y = Ae^{-(1/2)t^2} + 5$
- 19.3.4.  $y = Ae^{-e^t} - 2$



- 19.3.5.**  $y = Ae^t - t^2 - 2t - 2$   
**19.3.6.**  $y = Ae^{-t/2} + t - 2$   
**19.3.7.**  $y = At^2 - \frac{1}{3t}$   
**19.3.8.**  $y = \frac{c}{t} + \frac{2}{3}\sqrt{t}$   
**19.3.9.**  $y = A \cos t + \sin t$   
**19.3.10.**  $y = \frac{A}{\sec t + \tan t} + 1 - \frac{t}{\sec t + \tan t}$   
**19.4.1.**  $y(1) \approx 1.355$   
**19.4.2.**  $y(1) \approx 40.31$   
**19.4.3.**  $y(1) \approx 1.05$   
**19.4.4.**  $y(1) \approx 2.30$   
**19.5.4.**  $\frac{\omega + 1}{2\omega}e^{\omega t} + \frac{\omega - 1}{2\omega}e^{-\omega t}$   
**19.5.5.**  $2 \cos(3t) + 5 \sin(3t)$   
**19.5.6.**  $-(1/4)e^{-5t} + (5/4)e^{-t}$   
**19.5.7.**  $-2e^{-3t} + 2e^{4t}$   
**19.5.8.**  $5e^{-6t} + 20te^{-6t}$   
**19.5.9.**  $(16t - 3)e^{4t}$   
**19.5.10.**  $-2 \cos(\sqrt{5}t) + \sqrt{5} \sin(\sqrt{5}t)$   
**19.5.11.**  $-\sqrt{2} \cos t + \sqrt{2} \sin t$   
**19.5.12.**  $e^{-6t} (4 \cos t + 24 \sin t)$   
**19.5.13.**  $2e^{-3t} \sin(3t)$   
**19.5.14.**  $2 \cos(2t - \pi/6)$   
**19.5.15.**  $5\sqrt{2} \cos(10t - \pi/4)$   
**19.5.16.**  $\sqrt{2}e^{-2t} \cos(3t - \pi/4)$   
**19.5.17.**  $5e^{4t} \cos(3t + \arcsin(4/5))$   
**19.5.18.**  $(2 \cos(5t) + \sin(5t))e^{-2t}$   
**19.5.19.**  $-(1/2)e^{-2t} \sin(2t)$   
**19.6.1.**  $Ae^{5t} + Bte^{5t} + (6/169) \cos t - (5/338) \sin t$   
**19.6.2.**  $Ae^{-\sqrt{2}t} + Bte^{-\sqrt{2}t} + 5$   
**19.6.3.**  $A \cos(4t) + B \sin(4t) + (1/2)t^2 + (3/16)t - 5/16$   
**19.6.4.**  $A \cos(\sqrt{2}t) + B \sin(\sqrt{2}t) - (\cos(5t) + \sin(5t))/23$   
**19.6.5.**  $e^t(A \cos t + B \sin t) + e^{2t}/2$   
**19.6.6.**  $Ae^{\sqrt{6}t} + Be^{-\sqrt{6}t} + 2 - t/3 - e^{-t}/5$   
**19.6.7.**  $Ae^{-3t} + Be^{2t} - (1/5)te^{-3t}$   
**19.6.8.**  $Ae^t + Be^{3t} + (1/2)te^{3t}$   
**19.6.9.**  $A \cos(4t) + B \sin(4t) + (1/8)t \sin(4t)$   
**19.6.10.**  $A \cos(3t) + B \sin(3t) - (1/2)t \cos(3t)$   
**19.6.11.**  $Ae^{-6t} + Bte^{-6t} + 3t^2e^{-6t}$   
**19.6.12.**  $Ae^{4t} + Bte^{4t} - t^2e^{4t}$   
**19.6.13.**  $Ae^{-t} + Be^{-5t} + (4/5)$   
**19.6.14.**  $Ae^{4t} + Be^{-3t} + (1/144) - (t/12)$   
**19.6.15.**  $A \cos(\sqrt{5}t) + B \sin(\sqrt{5}t) + 8 \sin(2t)$   
**19.6.16.**  $Ae^{2t} + Be^{-2t} + te^{2t}$   
**19.6.17.**  $4e^t + e^{-t} - 3t - 5$   
**19.6.18.**  $-(4/27) \sin(3t) + (4/9)t$   
**19.6.19.**  $e^{-6t}(2 \cos t + 20 \sin t) + 2e^{-4t}$   
**19.6.20.**  $\left(-\frac{23}{325} \cos(3t) + \frac{592}{975} \sin(3t)\right) + \frac{23}{325} \cos t - \frac{11}{325} \sin t$   
**19.6.21.**  $e^{-2t}(A \sin(5t) + B \cos(5t)) + 8 \sin(2t) + 25 \cos(2t)$   
**19.6.22.**  $e^{-2t}(A \sin(2t) + B \cos(2t)) + (14/195) \sin t - (8/195) \cos t$   
**19.7.1.**  $A \sin(t) + B \cos(t) - \cos t \ln |\sec t + \tan t|$   
**19.7.2.**  $A \sin(t) + B \cos(t) + \frac{1}{5}e^{2t}$   
**19.7.3.**  $A \sin(2t) + B \cos(2t) + \cos t - \sin t \cos t \ln |\sec t + \tan t|$

$$\begin{aligned}
 \mathbf{19.7.4.} \quad & A \sin(2t) + B \cos(2t) + \\
 & \frac{1}{2} \sin(2t) \sin^2(t) + \\
 & \frac{1}{2} \sin(2t) \ln |\cos t| - \frac{t}{2} \cos(2t) + \\
 & \frac{1}{4} \sin(2t) \cos(2t)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{19.7.5.} \quad & A e^{2t} + B e^{-3t} + \frac{t^3}{15} e^{2t} - \\
 & \left( \frac{t^2}{5} - \frac{2t}{25} + \frac{2}{125} \right) \frac{e^{2t}}{5}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{19.7.6.} \quad & A e^t \sin t + B e^t \cos t - \\
 & e^t \cos t \ln |\sec t + \tan t|
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{19.7.7.} \quad & A e^t \sin t + B e^t \cos t - \frac{1}{10} \cos t (\cos^3 t + \\
 & 3 \sin^3 t - 2 \cos t - \sin t) + \\
 & \frac{1}{10} \sin t (\sin^3 t - 3 \cos^3 t - 2 \sin t + \\
 & \cos t) = \frac{1}{10} \cos(2t) - \frac{1}{20} \sin(2t)
 \end{aligned}$$