

# Index

## A

absolute extremum, 119  
acceleration  
  normal component, 345  
  tangential component, 345  
acceleration vector, 345  
algebraic precedence, 507  
alternating harmonic series, 273  
anti-parallel vectors, 309  
antiderivative, 155  
arc length, 232, 339  
arc length parameterization, 340  
arccosine, 96  
arcsine, 94  
area  
  between curves, 191  
  under a curve, 421  
  under curve, 151  
asymptote, 23, 115  
average, 405  
average height, 385

## B

bell curve, 226  
binormal, 344, 437  
bounded function, 54

## C

cardioid, 242  
Cartesian coordinates, 241, 323

Cauchy Principal Value, 220  
center of mass, 405  
center of mass, 215  
centroid, 216, 402  
chain rule, 67  
characteristic polynomial, 470  
chord, 32  
circle  
  area, 507  
  circumference, 507  
  equation of, 21, 508  
  unit, 21  
Clairaut's Theorem, 372  
closed paths, 427  
completing the square, 507  
composition of functions, 27, 45, 67  
concave down, 114  
concave up, 114  
cone  
  lateral area, 508  
  surface area, 508  
  volume, 508  
conservative vector field, 426  
continuous, 55, 355  
convergent sequence, 259  
convergent series, 264  
coordinates  
  Cartesian, 241, 323  
  converting rectangular to polar, 242  
  cylindrical, 323  
  polar, 241, 323  
  rectangular, 241, 323  
  spherical, 325

## 512 Index

cosines  
  law of, 307, 508  
critical value, 108  
cross product, 313  
cumulative distribution function, 226  
curvature, 342  
curvature formula, 342, 343  
curve sketching, 107  
cycloid, 251  
cylinder  
  lateral area, 508  
  surface area, 508  
  volume, 508  
cylindrical coordinates, 407  
cylindrical coordinates, 323

## D

definite integral, 159  
del ( $\nabla$ ), 367  
dependent variable, 24  
derivative, 34, 48  
  dot notation, 132  
  Leibniz notation, 49  
  second, 112, 113  
difference quotient, 32  
differentiable, 55, 362  
differential, 144  
differential equation, 455  
  first order, 456  
  particular solution, 458  
  separable, 458  
directional derivative, 367  
discrete probability, 223  
discriminant, 373  
displacement vector, 303  
divergence test, 266  
Divergence Theorem, 450  
divergent sequence, 259  
divergent series, 264  
domain, 23  
dot notation, 132  
dot product, 308  
double integral, 386

## E

ellipse  
  area, 507  
  equation of, 508  
ellipsoid, 239  
error estimate, 185

escape velocity, 222  
Euler's Method, 466  
exp function, 85  
expected value, 224  
exponential distribution, 227  
exponential function, 82  
Extreme Value Theorem, 121

## F

Fermat's Theorem, 108  
flux, 443  
frustum, 235  
Fubini's Theorem, 389  
function, 22  
  bounded, 54  
  differentiable, 55  
  implicit, 90  
  linear, 22  
  of two variables, 356  
  rational, 180  
  unbounded, 54  
function composition, 27  
Fundamental Theorem of Algebra, 111  
Fundamental Theorem of Calculus, 153

## G

Gabriel's horn, 223, 238  
geometric series, 264  
global extremum, 119  
gradient, 367  
greatest integer, 110

## H

harmonic series, 267  
  alternating, 273  
Hooke's Law, 211  
hyperbolic cosine, 102  
hyperbolic sine, 102  
hypercycloid, 252  
hypocycloid, 252

## I

implicit differentiation, 89, 365  
implicit function, 90  
improper integral, 219  
  convergent, 219  
  diverges, 219  
incompressible, 436  
indefinite integral, 159

independent variable, 24  
 inflection point, 114  
 initial condition, 456  
 initial value problem  
   first order, 456  
 integral  
   improper, 219  
   indefinite, 159  
   of  $\sec x$ , 175  
   of  $\sec^3 x$ , 175  
   properties of, 163  
 integral sign, 155  
 integral test, 270  
 integration  
   by parts, 176  
 Intermediate Value Theorem, 55  
 interval of convergence, 284  
 inverse function, 82  
 inverse sine, 94  
 involute, 252

**J**

Jacobian, 416  
 judicious guessing, 473

**K**

kinetic energy, 221

**L**

L'Hôpital's Rule, 98  
 Lagrange multipliers, 378  
 lateral area of a cone, 130  
 law of cosines, 307, 508  
 law of sines, 508  
 Leibniz notation, 49  
 level curve, 351  
 level set, 352  
 level surface, 352  
 limit, 41  
 limit at infinity, 98  
 limit of a sequence, 259  
 line integral, 421  
 linear approximation, 143  
 linearity of the derivative, 60  
 local extremum, 107  
 local maximum, 107  
 local minimum, 107  
 logarithm, 82  
 logarithmic function, 82

logistic equation, 460  
 long division of polynomials, 182

**M**

Maclaurin series, 288  
 mass, 405  
 mean, 224, 227  
 Mean Value Theorem, 146  
 moment, 215, 400, 406

**N**

Newton, 210  
 Newton's law of cooling, 455  
 Newton's method, 139–141  
 normal, 317, 344  
 normal distribution, 232

**O**

one sided limit, 46  
 optimization, 119  
 orientable surface, 444  
 oriented curve, 429

**P**

$p$ -series, 270  
 parallel vectors, 309  
 parallelogram  
   and vector sum, 303  
   area of, 315, 402, 414  
 parametric equations, 250, 320, 330  
 partial fractions, 180  
 particular solution, 458  
 physicists, 132  
 point-slope formula, 508  
 polar coordinates, 241, 323  
 polynomial  
   of two variables, 356  
 power function, 57  
 power rule, 57, 89, 92  
 precedence  
   of algebraic operations, 507  
 probability density function, 225  
 product rule, 62, 63  
   generalized, 64  
 projection, 423  
   scalar, 423  
 projection of a vector, 309  
 properties of integrals, 163

**Q**

quadratic formula, 507  
 quotient rule, 64

**R**

radian measure, 73  
 radius of convergence, 284  
 random variable, 224  
 rational function, 66, 180  
 rectangular coordinates, 241, 323  
 related rates, 131  
 resonant frequency, 477  
 right hand rule, 316  
 Rolle's Theorem, 146

**S**

scalar multiplication, 304  
 scalar projection, 423  
 second derivative, 112, 113  
 separation of variables, 458  
 sequence, 258  
   bounded, 262  
   bounded above, 262  
   bounded below, 262  
   convergent, 259  
   decreasing, 262  
   divergent, 259  
   increasing, 262  
   monotonic, 262  
   non-decreasing, 262  
   non-increasing, 262  
   of partial sums, 264  
 series, 258  
    $p$ -series, 270  
   absolute convergence, 278  
   alternating harmonic, 273  
   conditional convergence, 278  
   convergent, 264  
   divergent, 264  
   geometric, 264  
   harmonic, 267  
   integral test, 270  
   interval of convergence, 284  
   Maclaurin, 288  
   radius of convergence, 284  
   Taylor, 290  
 Simpson's Rule, 187  
 sines  
   law of, 508  
 slope field, 468

sphere  
   surface area, 508  
   volume, 508  
 spherical coordinates, 325, 407  
 spiral of Archimedes, 243  
 squeeze theorem, 77  
 standard deviation, 229  
 standard normal distribution, 227  
 standard normal probability density  
   function, 226  
 steady state part of solution to d.e., 476  
 substitution, 411  
 subtend, 74  
 sum  
   of vectors, 303  
 sum rule, 61  
 surface area, 402

**T**

tangent line, 32  
 Taylor series, 290  
 Toricelli's trumpet, 223, 238  
 torque, 213  
 torus, 238  
 transcendental function, 73  
 transient part of solution to d.e., 476  
 Trapezoid Rule, 185  
 triangle inequality, 43, 302  
 trigonometric identities, 508

**U**

unbounded function, 54  
 undetermined coefficients, 473  
 uniform distribution, 226  
 uniform probability density function, 226  
 unit binormal, 344  
 unit circle, 21  
 unit normal, 344  
 unit vector, 310, 336

**V**

variance, 229  
 variation of parameters, 463, 477  
 vector, 302  
   anti-parallel, 309  
   cross product, 313  
   displacement, 303  
   function, 330  
   normal to a plane, 317

- parallel, 309
- projection, 309
- scalar multiplication, 304
- sum, 303
- unit, 310
- vector fields, 419
- velocity, 37
- velocity vector, 345

**W**

- witch of Agnesi, 66
- work, 209