
Single and Multivariable Calculus

Early Transcendentals



This work is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike License. To view a copy of this license, visit <http://creativecommons.org/licenses/by-nc-sa/3.0/> or send a letter to Creative Commons, 543 Howard Street, 5th Floor, San Francisco, California, 94105, USA. If you distribute this work or a derivative, include the history of the document.

This text was initially written by David Guichard. The single variable material in chapters 1–9 is a modification and expansion of notes written by Neal Koblitz at the University of Washington, who generously gave permission to use, modify, and distribute his work. New material has been added, and old material has been modified, so some portions now bear little resemblance to the original.

The book includes some exercises and examples from *Elementary Calculus: An Approach Using Infinitesimals*, by H. Jerome Keisler, available at <http://www.math.wisc.edu/~keisler/calc.html> under a Creative Commons license. In addition, the chapter on differential equations (in the multivariable version) and the section on numerical integration are largely derived from the corresponding portions of Keisler's book. Albert Schueller, Barry Balof, and Mike Wills have contributed additional material.

This copy of the text was compiled from source at 19:59 on 5/16/2017.

I will be glad to receive corrections and suggestions for improvement at guichard@whitman.edu.

*For Kathleen,
without whose encouragement
this book would not have
been written.*

Contents

1

Analytic Geometry	15
1.1 Lines	16
1.2 Distance Between Two Points; Circles	21
1.3 Functions	22
1.4 Shifts and Dilations	27

2

Instantaneous Rate of Change: The Derivative	31
2.1 The slope of a function	31
2.2 An example	36
2.3 Limits	38
2.4 The Derivative Function	48
2.5 Adjectives For Functions	53

3

Rules for Finding Derivatives 57

3.1	The Power Rule	57
3.2	Linearity of the Derivative	60
3.3	The Product Rule	62
3.4	The Quotient Rule	64
3.5	The Chain Rule	67

4

Transcendental Functions 73

4.1	Trigonometric Functions	73
4.2	The Derivative of $\sin x$	76
4.3	A hard limit	77
4.4	The Derivative of $\sin x$, continued	80
4.5	Derivatives of the Trigonometric Functions	81
4.6	Exponential and Logarithmic functions	82
4.7	Derivatives of the exponential and logarithmic functions	84
4.8	Implicit Differentiation	89
4.9	Inverse Trigonometric Functions	94
4.10	Limits revisited	97
4.11	Hyperbolic Functions	102

5

Curve Sketching 107

5.1	Maxima and Minima	107
5.2	The first derivative test	111
5.3	The second derivative test	112
5.4	Concavity and inflection points	113
5.5	Asymptotes and Other Things to Look For	115

6

Applications of the Derivative 119

6.1	Optimization	119
6.2	Related Rates	131
6.3	Newton's Method	139
6.4	Linear Approximations	143
6.5	The Mean Value Theorem	145

7

Integration 149

7.1	Two examples	149
7.2	The Fundamental Theorem of Calculus	153
7.3	Some Properties of Integrals	160

8

Techniques of Integration 165

8.1	Substitution	166
8.2	Powers of sine and cosine	171
8.3	Trigonometric Substitutions	173
8.4	Integration by Parts	176
8.5	Rational Functions	180
8.6	Numerical Integration	184
8.7	Additional exercises	189

9

Applications of Integration 191

9.1	Area between curves	191
9.2	Distance, Velocity, Acceleration	196
9.3	Volume	199
9.4	Average value of a function	206
9.5	Work	209
9.6	Center of Mass	213
9.7	Kinetic energy; improper integrals	219
9.8	Probability	223
9.9	Arc Length	232
9.10	Surface Area	234

10

Polar Coordinates, Parametric Equations 241

10.1	Polar Coordinates	241
10.2	Slopes in polar coordinates	245
10.3	Areas in polar coordinates	247
10.4	Parametric Equations	251
10.5	Calculus with Parametric Equations	253

11

Sequences and Series	257
11.1 Sequences	258
11.2 Series	264
11.3 The Integral Test	268
11.4 Alternating Series	273
11.5 Comparison Tests	275
11.6 Absolute Convergence	278
11.7 The Ratio and Root Tests	279
11.8 Power Series	282
11.9 Calculus with Power Series	285
11.10 Taylor Series	287
11.11 Taylor's Theorem	290
11.12 Additional exercises	296

12

Three Dimensions	299
12.1 The Coordinate System	299
12.2 Vectors	302
12.3 The Dot Product	307
12.4 The Cross Product	313
12.5 Lines and Planes	317
12.6 Other Coordinate Systems	323

13

Vector Functions	329
13.1 Space Curves	329
13.2 Calculus with vector functions	331
13.3 Arc length and curvature	339
13.4 Motion along a curve	345

14

Partial Differentiation 349

14.1	Functions of Several Variables	349
14.2	Limits and Continuity	353
14.3	Partial Differentiation	357
14.4	The Chain Rule	364
14.5	Directional Derivatives	366
14.6	Higher order derivatives	371
14.7	Maxima and minima	373
14.8	Lagrange Multipliers	378

15

Multiple Integration 385

15.1	Volume and Average Height	385
15.2	Double Integrals in Cylindrical Coordinates	395
15.3	Moment and Center of Mass	399
15.4	Surface Area	402
15.5	Triple Integrals	404
15.6	Cylindrical and Spherical Coordinates	407
15.7	Change of Variables	411

16

Vector Calculus 419

16.1	Vector Fields	419
16.2	Line Integrals	421
16.3	The Fundamental Theorem of Line Integrals	425
16.4	Green's Theorem	428
16.5	Divergence and Curl	433
16.6	Vector Functions for Surfaces	436
16.7	Surface Integrals	442
16.8	Stokes's Theorem	446
16.9	The Divergence Theorem	450

17

Differential Equations 455

17.1	First Order Differential Equations	456
17.2	First Order Homogeneous Linear Equations	460
17.3	First Order Linear Equations	463
17.4	Approximation	466
17.5	Second Order Homogeneous Equations	469
17.6	Second Order Linear Equations	473
17.7	Second Order Linear Equations, take two	477

A

Selected Answers 481

B

Useful Formulas 507

Index 511

Introduction

The emphasis in this course is on problems—doing calculations and story problems. To master problem solving one needs a tremendous amount of practice doing problems. The more problems you do the better you will be at doing them, as patterns will start to emerge in both the problems and in successful approaches to them. You will learn fastest and best if you devote some time to doing problems every day.

Typically the most difficult problems are story problems, since they require some effort before you can begin calculating. Here are some pointers for doing story problems:

1. Carefully read each problem twice before writing anything.
2. Assign letters to quantities that are described only in words; draw a diagram if appropriate.
3. Decide which letters are constants and which are variables. A letter stands for a constant if its value remains the same throughout the problem.
4. Using mathematical notation, write down what you know and then write down what you want to find.
5. Decide what category of problem it is (this might be obvious if the problem comes at the end of a particular chapter, but will not necessarily be so obvious if it comes on an exam covering several chapters).
6. Double check each step as you go along; don't wait until the end to check your work.
7. Use common sense; if an answer is out of the range of practical possibilities, then check your work to see where you went wrong.

Suggestions for Using This Text

1. Read the example problems carefully, filling in any steps that are left out (ask someone for help if you can't follow the solution to a worked example).
2. Later use the worked examples to study by covering the solutions, and seeing if you can solve the problems on your own.
3. Most exercises have answers in Appendix A; the availability of an answer is marked by " \Rightarrow " at the end of the exercise. In the pdf version of the full text, clicking on the arrow will take you to the answer. The answers should be used only as a final check on your work, not as a crutch. Keep in mind that sometimes an answer could be expressed in various ways that are algebraically equivalent, so don't assume that your answer is wrong just because it doesn't have exactly the same form as the answer in the back.
4. A few figures in the pdf and print versions of the book are marked with "(AP)" at the end of the caption. Clicking on this should open a related interactive applet or Sage worksheet in your web browser. Occasionally another link will do the same thing, like this example. (Note to users of a printed text: the words "this example" in the pdf file are blue, and are a link to a Sage worksheet.)