
Calculus

Early Transcendentals

This work is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike License. To view a copy of this license, visit <http://creativecommons.org/licenses/by-nc-sa/3.0/> or send a letter to Creative Commons, 543 Howard Street, 5th Floor, San Francisco, California, 94105, USA. If you distribute this work or a derivative, include the history of the document.

This text was initially written by David Guichard. The single variable material in chapters 1–9 is a modification and expansion of notes written by Neal Koblitz at the University of Washington, who generously gave permission to use, modify, and distribute his work. New material has been added, and old material has been modified, so some portions now bear little resemblance to the original.

The book includes some exercises and examples from *Elementary Calculus: An Approach Using Infinitesimals*, by H. Jerome Keisler, available at <http://www.math.wisc.edu/~keisler/calc.html> under a Creative Commons license. In addition, the chapter on differential equations is largely derived from the corresponding chapter in Keisler's book. Albert Schueller, Barry Balof, and Mike Wills have contributed additional material.

This copy of the text was compiled from source at 14:53 on 5/4/2012.

I will be glad to receive corrections and suggestions for improvement at guichard@whitman.edu.

*For Kathleen,
without whose encouragement
this book would not have
been written.*

Contents

1	
Analytic Geometry	1
1.1 Lines	2
1.2 Distance Between Two Points; Circles	7
1.3 Functions	8
1.4 Shifts and Dilations	13

2	
Instantaneous Rate of Change: The Derivative	17
2.1 The slope of a function	17
2.2 An example	22
2.3 Limits	24
2.4 The Derivative Function	34
2.5 Adjectives For Functions	39

Contents vii

6	
Applications of the Derivative	103
6.1 Optimization	103
6.2 Related Rates	115
6.3 Newton's Method	123
6.4 Linear Approximations	127
6.5 The Mean Value Theorem	129

7	
Integration	133
7.1 Two examples	133
7.2 The Fundamental Theorem of Calculus	137
7.3 Some Properties of Integrals	144

8	
Techniques of Integration	149
8.1 Substitution	150
8.2 Powers of sine and cosine	155
8.3 Trigonometric Substitutions	157
8.4 Integration by Parts	160
8.5 Rational Functions	163
8.6 Additional exercises	168

vi Contents

3	
Rules for Finding Derivatives	43
3.1 The Power Rule	43
3.2 Linearity of the Derivative	46
3.3 The Product Rule	48
3.4 The Quotient Rule	50
3.5 The Chain Rule	53

4	
Transcendental Functions	59
4.1 Trigonometric Functions	59
4.2 The Derivative of $\sin x$	62
4.3 A hard limit	63
4.4 The Derivative of $\sin x$, continued	65
4.5 Derivatives of the Trigonometric Functions	66
4.6 Exponential and Logarithmic functions	68
4.7 Derivatives of the exponential and logarithmic functions	70
4.8 Implicit Differentiation	75
4.9 Inverse Trigonometric Functions	79
4.10 Limits revisited	82
4.11 Hyperbolic Functions	87

5	
Curve Sketching	91
5.1 Maxima and Minima	91
5.2 The first derivative test	95
5.3 The second derivative test	96
5.4 Concavity and inflection points	97
5.5 Asymptotes and Other Things to Look For	99

viii Contents

9	
Applications of Integration	169
9.1 Area between curves	169
9.2 Distance, Velocity, Acceleration	174
9.3 Volume	177
9.4 Average value of a function	183
9.5 Work	187
9.6 Center of Mass	191
9.7 Kinetic energy; improper integrals	197
9.8 Probability	201
9.9 Arc Length	210
9.10 Surface Area	212

10	
Polar Coordinates, Parametric Equations	217
10.1 Polar Coordinates	217
10.2 Slopes in polar coordinates	221
10.3 Areas in polar coordinates	223
10.4 Parametric Equations	226
10.5 Calculus with Parametric Equations	229

11

Sequences and Series 233

11.1 Sequences 234

11.2 Series 240

11.3 The Integral Test 244

11.4 Alternating Series 249

11.5 Comparison Tests 251

11.6 Absolute Convergence 254

11.7 The Ratio and Root Tests 255

11.8 Power Series 258

11.9 Calculus with Power Series 261

11.10 Taylor Series 262

11.11 Taylor's Theorem 266

11.12 Additional exercises 272

12

Three Dimensions 275

12.1 The Coordinate System 275

12.2 Vectors 278

12.3 The Dot Product 283

12.4 The Cross Product 289

12.5 Lines and Planes 293

12.6 Other Coordinate Systems 299

13

Vector Functions 305

13.1 Space Curves 305

13.2 Calculus with vector functions 307

13.3 Arc length 315

13.4 Motion along a curve 321

14

Partial Differentiation 323

14.1 Functions of Several Variables 323

14.2 Limits and Continuity 327

14.3 Partial Differentiation 331

14.4 The Chain Rule 337

14.5 Directional Derivatives 340

14.6 Higher order derivatives 344

14.7 Maxima and minima 346

14.8 Lagrange Multipliers 351

15

Multiple Integration 357

15.1 Volume and Average Height 357

15.2 Double Integrals in Cylindrical Coordinates 367

15.3 Moment and Center of Mass 371

15.4 Surface Area 373

15.5 Triple Integrals 375

15.6 Cylindrical and Spherical Coordinates 378

15.7 Change of Variables 382

16

Vector Calculus 389

16.1 Vector Fields 389

16.2 Line Integrals 391

16.3 The Fundamental Theorem of Line Integrals 395

16.4 Green's Theorem 398

16.5 Divergence and Curl 403

16.6 Vector Equations of Surfaces 406

16.7 Surface Integrals 412

16.8 Stokes's Theorem 416

16.9 The Divergence Theorem 420

17

Differential Equations 425

17.1 First Order Differential Equations 426

17.2 First Order Homogeneous Linear Equations 430

17.3 First Order Linear Equations 433

17.4 Approximation 435

17.5 Second Order Homogeneous Equations 438

17.6 Second Order Linear Equations 441

17.7 Second Order Linear Equations, take two 446

A

Selected Answers 451

B

Useful Formulas 477

Index 481

Introduction

The emphasis in this course is on problems—doing calculations and story problems. To master problem solving one needs a tremendous amount of practice doing problems. The more problems you do the better you will be at doing them, as patterns will start to emerge in both the problems and in successful approaches to them. You will learn fastest and best if you devote some time to doing problems every day.

Typically the most difficult problems are story problems, since they require some effort before you can begin calculating. Here are some pointers for doing story problems:

1. Carefully read each problem twice before writing anything.
2. Assign letters to quantities that are described only in words; draw a diagram if appropriate.
3. Decide which letters are constants and which are variables. A letter stands for a constant if its value remains the same throughout the problem.
4. Using mathematical notation, write down what you know and then write down what you want to find.
5. Decide what category of problem it is (this might be obvious if the problem comes at the end of a particular chapter, but will not necessarily be so obvious if it comes on an exam covering several chapters).
6. Double check each step as you go along; don't wait until the end to check your work.
7. Use common sense; if an answer is out of the range of practical possibilities, then check your work to see where you went wrong.

Suggestions for Using This Text

1. Read the example problems carefully, filling in any steps that are left out (ask someone for help if you can't follow the solution to a worked example).
2. Later use the worked examples to study by covering the solutions, and seeing if you can solve the problems on your own.
3. Most exercises have answers in Appendix A; the availability of an answer is marked by " \Rightarrow " at the end of the exercise. In the pdf version of the full text, clicking on the arrow will take you to the answer. The answers should be used only as a final check on your work, not as a crutch. Keep in mind that sometimes an answer could be expressed in various ways that are algebraically equivalent, so don't assume that your answer is wrong just because it doesn't have exactly the same form as the answer in the back.
4. A few figures in the book are marked with "(AP)" at the end of the caption. Clicking on this should open a related Java applet or Sage worksheet in your web browser. Occasionally another link will do the same thing, like this example. (Note to users of a printed text: the words "this example" in the pdf file are blue, and are a link to a Sage worksheet.)