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# Calculus

*Early Transcendentals*

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This text was initially written by David Guichard. The single variable material in chapters 1–9 is a modification and expansion of notes written by Neal Koblitz at the University of Washington, who generously gave permission to use, modify, and distribute his work. New material has been added, and old material has been modified, so some portions now bear little resemblance to the original.

The book includes some exercises and examples from *Elementary Calculus: An Approach Using Infinitesimals*, by H. Jerome Keisler, available at <http://www.math.wisc.edu/~keisler/calc.html> under a Creative Commons license. In addition, the chapter on differential equations is largely derived from the corresponding chapter in Keisler's book. Albert Schueller, Barry Balof, and Mike Wills have contributed additional material.

This copy of the text was compiled from source at 14:53 on 5/4/2012.

I will be glad to receive corrections and suggestions for improvement at [guichard@whitman.edu](mailto:guichard@whitman.edu).

*For Kathleen,  
without whose encouragement  
this book would not have  
been written.*



# Contents

## 1

<b>Analytic Geometry</b>	<b>1</b>
1.1 Lines . . . . .	2
1.2 Distance Between Two Points; Circles . . . . .	7
1.3 Functions . . . . .	8
1.4 Shifts and Dilations . . . . .	13

## 2

<b>Instantaneous Rate of Change: The Derivative</b>	<b>17</b>
2.1 The slope of a function . . . . .	17
2.2 An example . . . . .	22
2.3 Limits . . . . .	24
2.4 The Derivative Function . . . . .	34
2.5 Adjectives For Functions . . . . .	39

## 3

---

### Rules for Finding Derivatives 43

<b>3.1</b>	The Power Rule . . . . .	43
<b>3.2</b>	Linearity of the Derivative . . . . .	46
<b>3.3</b>	The Product Rule . . . . .	48
<b>3.4</b>	The Quotient Rule . . . . .	50
<b>3.5</b>	The Chain Rule . . . . .	53

## 4

---

### Transcendental Functions 59

<b>4.1</b>	Trigonometric Functions . . . . .	59
<b>4.2</b>	The Derivative of $\sin x$ . . . . .	62
<b>4.3</b>	A hard limit . . . . .	63
<b>4.4</b>	The Derivative of $\sin x$ , continued . . . . .	65
<b>4.5</b>	Derivatives of the Trigonometric Functions . . . . .	66
<b>4.6</b>	Exponential and Logarithmic functions . . . . .	68
<b>4.7</b>	Derivatives of the exponential and logarithmic functions . . . . .	70
<b>4.8</b>	Implicit Differentiation . . . . .	75
<b>4.9</b>	Inverse Trigonometric Functions . . . . .	79
<b>4.10</b>	Limits revisited . . . . .	82
<b>4.11</b>	Hyperbolic Functions . . . . .	87

## 5

---

### Curve Sketching 91

<b>5.1</b>	Maxima and Minima . . . . .	91
<b>5.2</b>	The first derivative test . . . . .	95
<b>5.3</b>	The second derivative test . . . . .	96
<b>5.4</b>	Concavity and inflection points . . . . .	97
<b>5.5</b>	Asymptotes and Other Things to Look For . . . . .	99

## 6

---

### Applications of the Derivative 103

<b>6.1</b>	Optimization . . . . .	103
<b>6.2</b>	Related Rates . . . . .	115
<b>6.3</b>	Newton's Method . . . . .	123
<b>6.4</b>	Linear Approximations . . . . .	127
<b>6.5</b>	The Mean Value Theorem . . . . .	129

## 7

---

### Integration 133

<b>7.1</b>	Two examples . . . . .	133
<b>7.2</b>	The Fundamental Theorem of Calculus . . . . .	137
<b>7.3</b>	Some Properties of Integrals . . . . .	144

## 8

---

### Techniques of Integration 149

<b>8.1</b>	Substitution . . . . .	150
<b>8.2</b>	Powers of sine and cosine . . . . .	155
<b>8.3</b>	Trigonometric Substitutions . . . . .	157
<b>8.4</b>	Integration by Parts . . . . .	160
<b>8.5</b>	Rational Functions . . . . .	163
<b>8.6</b>	Additional exercises . . . . .	168

## 9

---

### Applications of Integration 169

9.1	Area between curves . . . . .	169
9.2	Distance, Velocity, Acceleration . . . . .	174
9.3	Volume . . . . .	177
9.4	Average value of a function . . . . .	183
9.5	Work . . . . .	187
9.6	Center of Mass . . . . .	191
9.7	Kinetic energy; improper integrals . . . . .	197
9.8	Probability . . . . .	201
9.9	Arc Length . . . . .	210
9.10	Surface Area . . . . .	212

## 10

---

### Polar Coordinates, Parametric Equations 217

10.1	Polar Coordinates . . . . .	217
10.2	Slopes in polar coordinates . . . . .	221
10.3	Areas in polar coordinates . . . . .	223
10.4	Parametric Equations . . . . .	226
10.5	Calculus with Parametric Equations . . . . .	229



# 11

---

<b>Sequences and Series</b>	<b>233</b>
11.1 Sequences . . . . .	234
11.2 Series . . . . .	240
11.3 The Integral Test . . . . .	244
11.4 Alternating Series . . . . .	249
11.5 Comparison Tests . . . . .	251
11.6 Absolute Convergence . . . . .	254
11.7 The Ratio and Root Tests . . . . .	255
11.8 Power Series . . . . .	258
11.9 Calculus with Power Series . . . . .	261
11.10 Taylor Series . . . . .	262
11.11 Taylor's Theorem . . . . .	266
11.12 Additional exercises . . . . .	272

# 12

---

<b>Three Dimensions</b>	<b>275</b>
12.1 The Coordinate System . . . . .	275
12.2 Vectors . . . . .	278
12.3 The Dot Product . . . . .	283
12.4 The Cross Product . . . . .	289
12.5 Lines and Planes . . . . .	293
12.6 Other Coordinate Systems . . . . .	299

# 13

---

<b>Vector Functions</b>	<b>305</b>
13.1 Space Curves . . . . .	305
13.2 Calculus with vector functions . . . . .	307
13.3 Arc length . . . . .	315
13.4 Motion along a curve . . . . .	321

## 14

---

### Partial Differentiation 323

14.1	Functions of Several Variables . . . . .	323
14.2	Limits and Continuity . . . . .	327
14.3	Partial Differentiation . . . . .	331
14.4	The Chain Rule . . . . .	337
14.5	Directional Derivatives . . . . .	340
14.6	Higher order derivatives . . . . .	344
14.7	Maxima and minima . . . . .	346
14.8	Lagrange Multipliers . . . . .	351

## 15

---

### Multiple Integration 357

15.1	Volume and Average Height . . . . .	357
15.2	Double Integrals in Cylindrical Coordinates . . . . .	367
15.3	Moment and Center of Mass . . . . .	371
15.4	Surface Area . . . . .	373
15.5	Triple Integrals . . . . .	375
15.6	Cylindrical and Spherical Coordinates . . . . .	378
15.7	Change of Variables . . . . .	382

## 16

---

### Vector Calculus 389

16.1	Vector Fields . . . . .	389
16.2	Line Integrals . . . . .	391
16.3	The Fundamental Theorem of Line Integrals . . . . .	395
16.4	Green's Theorem . . . . .	398
16.5	Divergence and Curl . . . . .	403
16.6	Vector Equations of Surfaces . . . . .	406
16.7	Surface Integrals . . . . .	412
16.8	Stokes's Theorem . . . . .	416
16.9	The Divergence Theorem . . . . .	420

# 17

## Differential Equations 425

- 17.1 First Order Differential Equations . . . . . 426
- 17.2 First Order Homogeneous Linear Equations . . . . . 430
- 17.3 First Order Linear Equations . . . . . 433
- 17.4 Approximation . . . . . 435
- 17.5 Second Order Homogeneous Equations . . . . . 438
- 17.6 Second Order Linear Equations . . . . . 441
- 17.7 Second Order Linear Equations, take two . . . . . 446

# A

## Selected Answers 451

# B

## Useful Formulas 477

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## Index 481



# Introduction

The emphasis in this course is on problems—doing calculations and story problems. To master problem solving one needs a tremendous amount of practice doing problems. The more problems you do the better you will be at doing them, as patterns will start to emerge in both the problems and in successful approaches to them. You will learn fastest and best if you devote some time to doing problems every day.

Typically the most difficult problems are story problems, since they require some effort before you can begin calculating. Here are some pointers for doing story problems:

1. Carefully read each problem twice before writing anything.
2. Assign letters to quantities that are described only in words; draw a diagram if appropriate.
3. Decide which letters are constants and which are variables. A letter stands for a constant if its value remains the same throughout the problem.
4. Using mathematical notation, write down what you know and then write down what you want to find.
5. Decide what category of problem it is (this might be obvious if the problem comes at the end of a particular chapter, but will not necessarily be so obvious if it comes on an exam covering several chapters).
6. Double check each step as you go along; don't wait until the end to check your work.
7. Use common sense; if an answer is out of the range of practical possibilities, then check your work to see where you went wrong.

### Suggestions for Using This Text

1. Read the example problems carefully, filling in any steps that are left out (ask someone for help if you can't follow the solution to a worked example).
2. Later use the worked examples to study by covering the solutions, and seeing if you can solve the problems on your own.
3. Most exercises have answers in Appendix A; the availability of an answer is marked by " $\Rightarrow$ " at the end of the exercise. In the pdf version of the full text, clicking on the arrow will take you to the answer. The answers should be used only as a final check on your work, not as a crutch. Keep in mind that sometimes an answer could be expressed in various ways that are algebraically equivalent, so don't assume that your answer is wrong just because it doesn't have exactly the same form as the answer in the back.
4. A few figures in the book are marked with "(AP)" at the end of the caption. Clicking on this should open a related Java applet or Sage worksheet in your web browser. Occasionally another link will do the same thing, like this example. (Note to users of a printed text: the words "this example" in the pdf file are blue, and are a link to a Sage worksheet.)