B

Useful Formulas

Algebra

Remember that the common algebraic operations have **precedences** relative to each other: for example, multiplication and division take precedence over addition and subtraction, but are "tied" with each other. In the case of ties, work left to right. This means, for example, that $1/2x$ means $(1/2)x$: do the division, then the multiplication in left to right order. It sometimes is a good idea to use more parentheses than strictly necessary, for clarity, but it is also a bad idea to use too many parentheses.

Completing the square: $x^2 + bx + c = (x + \frac{b}{2})^2 - \frac{b^2}{4} + c$.

Quadratic formula: the roots of $ax^2 + bx + c$ are $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Exponent rules:

$$a^b \cdot a^c = a^{b+c}$$
$$\frac{a^b}{a^c} = a^{b-c}$$
$$(a^b)^c = a^{bc}$$
$$a^{1/b} = \sqrt[b]{a}$$

Geometry

Circle: circumference $= 2\pi r$, area $= \pi r^2$.

Ellipse: area $= \pi ab$, where $2a$ and $2b$ are the lengths of the axes of the ellipse.

Sphere: vol $= 4\pi r^3/3$, surface area $= 4\pi r^2$.

Cylinder: vol $= \pi r^2 h$, lateral area $= 2\pi rh$, total surface area $= 2\pi rh + 2\pi r^2$.

Cone: vol $= \pi r^2 h/3$, lateral area $= \pi r\sqrt{r^2 + h^2}$, total surface area $= \pi r\sqrt{r^2 + h^2} + \pi r^2$.

Analytic geometry

Point-slope formula for straight line through the point (x_0, y_0) with slope m: $y = y_0 + m(x - x_0)$.

Circle with radius r centered at (h, k): $(x - h)^2 + (y - k)^2 = r^2$.

Ellipse with axes on the x-axis and y-axis: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

Trigonometry

$\sin(\theta) = \text{opposite}/\text{hypotenuse}$

$\cos(\theta) = \text{adjacent}/\text{hypotenuse}$

$\tan(\theta) = \text{opposite}/\text{adjacent}$

$\sec(\theta) = 1/\cos(\theta)$

$\csc(\theta) = 1/\sin(\theta)$

$\cot(\theta) = 1/\tan(\theta)$

$\tan(\theta) = \sin(\theta)/\cos(\theta)$

$\cot(\theta) = \cos(\theta)/\sin(\theta)$

$\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right)$

$\cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right)$

$\sin(\theta + \pi) = -\sin(\theta)$

$\cos(\theta + \pi) = -\cos(\theta)$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos A$

Law of sines: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Sine of sum of angles: $\sin(x + y) = \sin x \cos y + \cos x \sin y$

Sine of double angle: $\sin(2x) = 2\sin x \cos x$

Sine of difference of angles: $\sin(x - y) = \sin x \cos y - \cos x \sin y$

Cosine of sum of angles: $\cos(x + y) = \cos x \cos y - \sin x \sin y$

Cosine of double angle: $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

Cosine of difference of angles: $\cos(x - y) = \cos x \cos y + \sin x \sin y$

Tangent of sum of angles: $\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

$\sin^2(\theta)$ and $\cos^2(\theta)$ formulas:

$$\sin^2(\theta) + \cos^2(\theta) = 1$$
$$\tan^2(\theta) + 1 = \sec^2(\theta)$$
$$\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}$$
$$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$$