Single and Multivariable Calculus

Early Transcendentals

This text was initially written by David Guichard. The single variable material in chapters 1–9 is a modification and expansion of notes written by Neal Koblitz at the University of Washington, who generously gave permission to use, modify, and distribute his work. New material has been added, and old material has been modified, so some portions now bear little resemblance to the original.

The book includes some exercises and examples from *Elementary Calculus: An Approach Using Infinitesimals*, by H. Jerome Keisler, available at http://www.math.wisc.edu/~keisler/calc.html under a Creative Commons license. In addition, the chapter on differential equations (in the multivariable version) and the section on numerical integration are largely derived from the corresponding portions of Keisler's book. Albert Schueller, Barry Balof, and Mike Wills have contributed additional material.

This copy of the text was compiled from source at 12:08 on 10/14/2018.

I will be glad to receive corrections and suggestions for improvement at guichard@whitman.edu.

For Kathleen,
without whose encouragement
this book would not have
been written.

Contents

6

Applications of the Derivative 119

7

Integration 149

8

Techniques of Integration 165

9

Applications of Integration 191

10

Polar Coordinates, Parametric Equations 241

11

Sequences and Series 257

12

Three Dimensions 299

13

Vector Functions 329

14

Partial Differentiation 349

15

Multiple Integration 385

16

Vector Calculus 419

17

Differential Equations 455

A

Selected Answers 481

B

Useful Formulas 507

Index 511

Introduction

The emphasis in this course is on problems—doing calculations and story problems. To master problem solving one needs a tremendous amount of practice doing problems. The more problems you do the better you will be at doing them, as patterns will start to emerge in both the problems and in successful approaches to them. You will learn fastest and best if you devote some time to doing problems every day.

Typically the most difficult problems are story problems, since they require some effort before you can begin calculating. Here are some pointers for doing story problems:

1. Carefully read each problem twice before writing anything.

2. Assign letters to quantities that are described only in words; draw a diagram if appropriate.

3. Decide which letters are constants and which are variables. A letter stands for a constant if its value remains the same throughout the problem.

4. Using mathematical notation, write down what you know and then write down what you want to find.

5. Decide what category of problem it is (this might be obvious if the problem comes at the end of a particular chapter, but will not necessarily be so obvious if it comes on an exam covering several chapters).

6. Double check each step as you go along; don't wait until the end to check your work.

7. Use common sense; if an answer is out of the range of practical possibilities, then check your work to see where you went wrong.

Suggestions for Using This Text

1. Read the example problems carefully, filling in any steps that are left out (ask someone for help if you can't follow the solution to a worked example).

2. Later use the worked examples to study by covering the solutions, and seeing if you can solve the problems on your own.

3. Most exercises have answers in Appendix A; the availability of an answer is marked by "⇒" at the end of the exercise. In the pdf version of the full text, clicking on the arrow will take you to the answer. The answers should be used only as a final check on your work, not as a crutch. Keep in mind that sometimes an answer could be expressed in various ways that are algebraically equivalent, so don't assume that your answer is wrong just because it doesn't have exactly the same form as the answer in the back.

4. A few figures in the pdf and print versions of the book are marked with "(AP)" at the end of the caption. Clicking on this should open a related interactive applet or Sage worksheet in your web browser. Occasionally another link will do the same thing, like this example. (Note to users of a printed text: the words "this example" in the pdf file are blue, and are a link to a Sage worksheet.)