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# Calculus

*Late Transcendentals*

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This text was initially written by David Guichard. The single variable material in chapters 1–9 is a modification and expansion of notes written by Neal Koblitz at the University of Washington, who generously gave permission to use, modify, and distribute his work. New material has been added, and old material has been modified, so some portions now bear little resemblance to the original.

The book includes some exercises and examples from *Elementary Calculus: An Approach Using Infinitesimals*, by H. Jerome Keisler, available at <http://www.math.wisc.edu/~keisler/calc.html> under a Creative Commons license. In addition, the chapter on differential equations is largely derived from the corresponding chapter in Keisler's book. Albert Schueller, Barry Balof, and Mike Wills have contributed additional material.

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I will be glad to receive corrections and suggestions for improvement at [guichard@whitman.edu](mailto:guichard@whitman.edu).

*For Kathleen,  
without whose encouragement  
this book would not have  
been written.*



# Contents

## **1**

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<b>Analytic Geometry</b>	<b>1</b>
1.1 Lines . . . . .	2
1.2 Distance Between Two Points; Circles . . . . .	7
1.3 Functions . . . . .	8
1.4 Shifts and Dilations . . . . .	13

## **2**

---

<b>Instantaneous Rate of Change: The Derivative</b>	<b>17</b>
2.1 The slope of a function . . . . .	17
2.2 An example . . . . .	22
2.3 Limits . . . . .	24
2.4 The Derivative Function . . . . .	34
2.5 Adjectives For Functions . . . . .	39

## 3

---

### Rules for Finding Derivatives 43

<b>3.1</b>	The Power Rule . . . . .	43
<b>3.2</b>	Linearity of the Derivative . . . . .	46
<b>3.3</b>	The Product Rule . . . . .	48
<b>3.4</b>	The Quotient Rule . . . . .	50
<b>3.5</b>	The Chain Rule . . . . .	53

## 4

---

### Trigonometric Functions 59

<b>4.1</b>	Trigonometric Functions . . . . .	59
<b>4.2</b>	The Derivative of $\sin x$ . . . . .	62
<b>4.3</b>	A hard limit . . . . .	63
<b>4.4</b>	The Derivative of $\sin x$ , continued . . . . .	65
<b>4.5</b>	Derivatives of the Trigonometric Functions . . . . .	66
<b>4.6</b>	Implicit Differentiation . . . . .	68
<b>4.7</b>	Limits revisited . . . . .	72

## 5

---

### Curve Sketching 77

<b>5.1</b>	Maxima and Minima . . . . .	77
<b>5.2</b>	The first derivative test . . . . .	81
<b>5.3</b>	The second derivative test . . . . .	82
<b>5.4</b>	Concavity and inflection points . . . . .	83
<b>5.5</b>	Asymptotes and Other Things to Look For . . . . .	85

## 6

---

### Applications of the Derivative 89

<b>6.1</b>	Optimization . . . . .	89
<b>6.2</b>	Related Rates . . . . .	101
<b>6.3</b>	Newton's Method . . . . .	109
<b>6.4</b>	Linear Approximations . . . . .	113
<b>6.5</b>	The Mean Value Theorem . . . . .	115

## 7

---

### Integration 119

<b>7.1</b>	Two examples . . . . .	119
<b>7.2</b>	The Fundamental Theorem of Calculus . . . . .	123
<b>7.3</b>	Some Properties of Integrals . . . . .	130
<b>7.4</b>	Substitution . . . . .	134

## 8

---

### Applications of Integration 141

<b>8.1</b>	Area between curves . . . . .	141
<b>8.2</b>	Distance, Velocity, Acceleration . . . . .	146
<b>8.3</b>	Volume . . . . .	149
<b>8.4</b>	Average value of a function . . . . .	155
<b>8.5</b>	Work . . . . .	158

## 9

---

### Transcendental Functions 163

<b>9.1</b>	Inverse functions . . . . .	163
<b>9.2</b>	The natural logarithm . . . . .	169
<b>9.3</b>	The exponential function . . . . .	173
<b>9.4</b>	Other bases . . . . .	176
<b>9.5</b>	Inverse Trigonometric Functions . . . . .	180
<b>9.6</b>	Hyperbolic Functions . . . . .	183

## 10

---

### Techniques of Integration 189

<b>10.1</b>	Powers of sine and cosine . . . . .	189
<b>10.2</b>	Trigonometric Substitutions . . . . .	191
<b>10.3</b>	Integration by Parts . . . . .	194
<b>10.4</b>	Rational Functions . . . . .	197
<b>10.5</b>	Additional exercises . . . . .	202

11

More Applications of Integration 203

11.1 Center of Mass . . . . . 203

11.2 Kinetic energy; improper integrals . . . . . 209

11.3 Probability . . . . . 213

11.4 Arc Length . . . . . 222

11.5 Surface Area . . . . . 224

12

Polar Coordinates, Parametric Equations 229

12.1 Polar Coordinates . . . . . 229

12.2 Slopes in polar coordinates . . . . . 233

12.3 Areas in polar coordinates . . . . . 235

12.4 Parametric Equations . . . . . 238

12.5 Calculus with Parametric Equations . . . . . 241

13

Sequences and Series 245

13.1 Sequences . . . . . 246

13.2 Series . . . . . 252

13.3 The Integral Test . . . . . 256

13.4 Alternating Series . . . . . 261

13.5 Comparison Tests . . . . . 263

13.6 Absolute Convergence . . . . . 266

13.7 The Ratio and Root Tests . . . . . 267

13.8 Power Series . . . . . 270

13.9 Calculus with Power Series . . . . . 273

13.10 Taylor Series . . . . . 274

13.11 Taylor’s Theorem . . . . . 278

13.12 Additional exercises . . . . . 284

## A

---

Selected Answers	287
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## B

---

Useful Formulas	303
-----------------	-----

---

Index	307
-------	-----



# Introduction

The emphasis in this course is on problems—doing calculations and story problems. To master problem solving one needs a tremendous amount of practice doing problems. The more problems you do the better you will be at doing them, as patterns will start to emerge in both the problems and in successful approaches to them. You will learn fastest and best if you devote some time to doing problems every day.

Typically the most difficult problems are story problems, since they require some effort before you can begin calculating. Here are some pointers for doing story problems:

1. Carefully read each problem twice before writing anything.
2. Assign letters to quantities that are described only in words; draw a diagram if appropriate.
3. Decide which letters are constants and which are variables. A letter stands for a constant if its value remains the same throughout the problem.
4. Using mathematical notation, write down what you know and then write down what you want to find.
5. Decide what category of problem it is (this might be obvious if the problem comes at the end of a particular chapter, but will not necessarily be so obvious if it comes on an exam covering several chapters).
6. Double check each step as you go along; don't wait until the end to check your work.
7. Use common sense; if an answer is out of the range of practical possibilities, then check your work to see where you went wrong.

## Suggestions for Using This Text

1. Read the example problems carefully, filling in any steps that are left out (ask someone if you can't follow the solution to a worked example).
2. Later use the worked examples to study by covering the solutions, and seeing if you can solve the problems on your own.
3. Most exercises have answers in Appendix A; the availability of an answer is marked by " $\Rightarrow$ " at the end of the exercise. In the pdf version of the full text, clicking on the arrow will take you to the answer. The answers should be used only as a final check on your work, not as a crutch. Keep in mind that sometimes an answer could be expressed in various ways that are algebraically equivalent, so don't assume that your answer is wrong just because it doesn't have exactly the same form as the answer in the back.
4. A few figures in the book are marked with "(JA)" at the end of the caption. Clicking on this should open a related Java applet or Sage worksheet in your web browser. Occasionally another link will do the same thing, like this example. (Note to users of a printed text: the words "this example" in the pdf file are blue, and are a link to a Sage worksheet.)