Single Variable Calculus

Late Transcendentals

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This text was initially written by David Guichard. The single variable material in chapters 1–9 is a modification and expansion of notes written by Neal Koblitz at the University of Washington, who generously gave permission to use, modify, and distribute his work. New material has been added, and old material has been modified, so some portions now bear little resemblance to the original.

The book includes some exercises and examples from *Elementary Calculus: An Approach Using Infinitesimals*, by H. Jerome Keisler, available at http://www.math.wisc.edu/~keisler/calc.html under a Creative Commons license. In addition, the chapter on differential equations (in the multivariable version) and the section on numerical integration are largely derived from the corresponding portions of Keisler's book.

Some exercises are from the OpenStax Calculus books, available free at
https://openstax.org/subjects/math.

Albert Schueller, Barry Balof, and Mike Wills have contributed additional material.

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The current version of the text is available at
https://www.whitman.edu/mathematics/multivariable/.

I will be glad to receive corrections and suggestions for improvement at guichard@whitman.edu.

For Kathleen,
without whose encouragement
this book would not have
been written.

Contents

1

Analytic Geometry **13**

1.1 Lines 14
1.2 Distance Between Two Points; Circles 19
1.3 Functions 20
1.4 Shifts and Dilations 25

2

Instantaneous Rate of Change: The Derivative **29**

2.1 The slope of a function 29
2.2 An example 34
2.3 Limits 36
2.4 The Derivative Function 46
2.5 Properties of Functions 51

3

Rules for Finding Derivatives 55

3.1 The Power Rule 55
3.2 Linearity of the Derivative 58
3.3 The Product Rule 60
3.4 The Quotient Rule 62
3.5 The Chain Rule 65

4

Trigonometric Functions 71

4.1 Trigonometric Functions 71
4.2 The Derivative of $\sin x$ 74
4.3 A hard limit 75
4.4 The Derivative of $\sin x$, continued 78
4.5 Derivatives of the Trigonometric Functions 79
4.6 Implicit Differentiation 80
4.7 Limits revisited 84

5

Curve Sketching 89

5.1 Maxima and Minima 89
5.2 The first derivative test 93
5.3 The second derivative test 95
5.4 Concavity and inflection points 96
5.5 Asymptotes and Other Things to Look For 98

6

Applications of the Derivative 101

6.1 Optimization 101
6.2 Related Rates 113
6.3 Newton's Method 121
6.4 Linear Approximations 125
6.5 The Mean Value Theorem 127

7

Integration 131

7.1 Two examples 131
7.2 The Fundamental Theorem of Calculus 135
7.3 Some Properties of Integrals 142
7.4 Substitution 146

8

Applications of Integration 153

8.1 Area between curves 153
8.2 Distance, Velocity, Acceleration 158
8.3 Volume 161
8.4 Average value of a function 168
8.5 Work 171

9

Transcendental Functions 177

9.1 Inverse functions 177
9.2 The natural logarithm 183
9.3 The exponential function 187
9.4 Other bases 190
9.5 Inverse Trigonometric Functions 194
9.6 Hyperbolic Functions 197

10

Techniques of Integration 203

10.1 Powers of sine and cosine 203
10.2 Trigonometric Substitutions 205
10.3 Integration by Parts 208
10.4 Rational Functions 212
10.5 Numerical Integration 216
10.6 Additional exercises 221

11

More Applications of Integration 223

11.1 Center of Mass 223
11.2 Kinetic energy; improper integrals 229
11.3 Probability 233
11.4 Arc Length 242
11.5 Surface Area 244

12

Polar Coordinates, Parametric Equations 251

12.1 Polar Coordinates 251
12.2 Slopes in polar coordinates 255
12.3 Areas in polar coordinates 257
12.4 Parametric Equations 260
12.5 Calculus with Parametric Equations 263

13

Sequences and Series 267

13.1 Sequences 268
13.2 Series 274
13.3 The Integral Test 278
13.4 Alternating Series 283
13.5 Comparison Tests 285
13.6 Absolute Convergence 288
13.7 The Ratio and Root Tests 289
13.8 Power Series 292
13.9 Calculus with Power Series 295
13.10 Taylor Series 297
13.11 Taylor's Theorem 300
13.12 Additional exercises 306

A

Selected Answers 309

B

Useful Formulas 325

Index 329

Introduction

The emphasis in this course is on problems—doing calculations and story problems. To master problem solving one needs a tremendous amount of practice doing problems. The more problems you do the better you will be at doing them, as patterns will start to emerge in both the problems and in successful approaches to them. You will learn fastest and best if you devote some time to doing problems every day.

Typically the most difficult problems are story problems, since they require some effort before you can begin calculating. Here are some pointers for doing story problems:

1. Carefully read each problem twice before writing anything.
2. Assign letters to quantities that are described only in words; draw a diagram if appropriate.
3. Decide which letters are constants and which are variables. A letter stands for a constant if its value remains the same throughout the problem.
4. Using mathematical notation, write down what you know and then write down what you want to find.
5. Decide what category of problem it is (this might be obvious if the problem comes at the end of a particular chapter, but will not necessarily be so obvious if it comes on an exam covering several chapters).
6. Double check each step as you go along; don't wait until the end to check your work.
7. Use common sense; if an answer is out of the range of practical possibilities, then check your work to see where you went wrong.

Suggestions for Using This Text

1. Read the example problems carefully, filling in any steps that are left out (ask someone for help if you can't follow the solution to a worked example).
2. Later use the worked examples to study by covering the solutions, and seeing if you can solve the problems on your own.
3. Most exercises have answers in Appendix A; the availability of an answer is marked by "⇒" at the end of the exercise. Clicking on the arrow will take you to the answer. The answers should be used only as a final check on your work, not as a crutch. Keep in mind that sometimes an answer could be expressed in various ways that are algebraically equivalent, so don't assume that your answer is wrong just because it doesn't have exactly the same form as the given answer.
4. A few figures in the pdf and print versions of the book are marked with "(AP)" at the end of the caption. Clicking on this in the pdf should open a related interactive applet or Sage worksheet in your web browser. Occasionally another link will do the same thing, like this example. (Note to users of a printed text: the words "this example" in the pdf file are blue, and are a link to a Sage worksheet.) In the html version of the text, these features appear in the text itself.