Single Variable Calculus

Late Transcendentals
For Kathleen,
without whose encouragement
this book would not have
been written.
# Contents

## 1 Analytic Geometry

1.1 Lines ........................................... 14
1.2 Distance Between Two Points; Circles .................. 19
1.3 Functions ...................................... 20
1.4 Shifts and Dilations ................................ 25

## 2 Instantaneous Rate of Change: The Derivative

2.1 The slope of a function ............................ 29
2.2 An example ...................................... 34
2.3 Limits ........................................... 36
2.4 The Derivative Function ......................... 46
2.5 Properties of Functions .......................... 51
## Rules for Finding Derivatives

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>The Power Rule</td>
<td>55</td>
</tr>
<tr>
<td>3.2</td>
<td>Linearity of the Derivative</td>
<td>58</td>
</tr>
<tr>
<td>3.3</td>
<td>The Product Rule</td>
<td>60</td>
</tr>
<tr>
<td>3.4</td>
<td>The Quotient Rule</td>
<td>62</td>
</tr>
<tr>
<td>3.5</td>
<td>The Chain Rule</td>
<td>65</td>
</tr>
</tbody>
</table>

## Trigonometric Functions

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>Trigonometric Functions</td>
<td>71</td>
</tr>
<tr>
<td>4.2</td>
<td>The Derivative of $\sin x$</td>
<td>74</td>
</tr>
<tr>
<td>4.3</td>
<td>A hard limit</td>
<td>75</td>
</tr>
<tr>
<td>4.4</td>
<td>The Derivative of $\sin x$, continued</td>
<td>78</td>
</tr>
<tr>
<td>4.5</td>
<td>Derivatives of the Trigonometric Functions</td>
<td>79</td>
</tr>
<tr>
<td>4.6</td>
<td>Implicit Differentiation</td>
<td>80</td>
</tr>
<tr>
<td>4.7</td>
<td>Limits revisited</td>
<td>84</td>
</tr>
</tbody>
</table>

## Curve Sketching

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>Maxima and Minima</td>
<td>89</td>
</tr>
<tr>
<td>5.2</td>
<td>The first derivative test</td>
<td>93</td>
</tr>
<tr>
<td>5.3</td>
<td>The second derivative test</td>
<td>95</td>
</tr>
<tr>
<td>5.4</td>
<td>Concavity and inflection points</td>
<td>96</td>
</tr>
<tr>
<td>5.5</td>
<td>Asymptotes and Other Things to Look For</td>
<td>98</td>
</tr>
</tbody>
</table>

## Applications of the Derivative

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td>Optimization</td>
<td>101</td>
</tr>
<tr>
<td>6.2</td>
<td>Related Rates</td>
<td>113</td>
</tr>
<tr>
<td>6.3</td>
<td>Newton’s Method</td>
<td>121</td>
</tr>
<tr>
<td>6.4</td>
<td>Linear Approximations</td>
<td>125</td>
</tr>
<tr>
<td>6.5</td>
<td>The Mean Value Theorem</td>
<td>127</td>
</tr>
</tbody>
</table>
# Integration

7.1 Two examples .................................. 131
7.2 The Fundamental Theorem of Calculus ........... 135
7.3 Some Properties of Integrals .................. 142
7.4 Substitution .................................. 146

# Applications of Integration

8.1 Area between curves .......................... 153
8.2 Distance, Velocity, Acceleration ............. 158
8.3 Volume ...................................... 161
8.4 Average value of a function .................. 168
8.5 Work ........................................ 171

# Transcendental Functions

9.1 Inverse functions ............................. 177
9.2 The natural logarithm .......................... 183
9.3 The exponential function ..................... 187
9.4 Other bases .................................. 190
9.5 Inverse Trigonometric Functions ............. 194
9.6 Hyperbolic Functions ........................ 197

# Techniques of Integration

10.1 Powers of sine and cosine .................... 203
10.2 Trigonometric Substitutions ................. 205
10.3 Integration by Parts .......................... 208
10.4 Rational Functions ........................... 212
10.5 Numerical Integration ........................ 216
10.6 Additional exercises .......................... 221
# Contents

11 More Applications of Integration 223
11.1 Center of Mass .......................... 223
11.2 Kinetic energy; improper integrals .......................... 229
11.3 Probability .............................. 233
11.4 Arc Length .............................. 242
11.5 Surface Area .............................. 244

12 Polar Coordinates, Parametric Equations 251
12.1 Polar Coordinates .......................... 251
12.2 Slopes in polar coordinates .......................... 255
12.3 Areas in polar coordinates .......................... 257
12.4 Parametric Equations .......................... 260
12.5 Calculus with Parametric Equations .......................... 263

13 Sequences and Series 267
13.1 Sequences .............................. 268
13.2 Series .............................. 274
13.3 The Integral Test .......................... 278
13.4 Alternating Series .......................... 283
13.5 Comparison Tests .......................... 285
13.6 Absolute Convergence .......................... 288
13.7 The Ratio and Root Tests .......................... 289
13.8 Power Series .............................. 292
13.9 Calculus with Power Series .......................... 295
13.10 Taylor Series .............................. 297
13.11 Taylor’s Theorem .......................... 300
13.12 Additional exercises .......................... 306
Introduction

The emphasis in this course is on problems—doing calculations and story problems. To master problem solving one needs a tremendous amount of practice doing problems. The more problems you do the better you will be at doing them, as patterns will start to emerge in both the problems and in successful approaches to them. You will learn fastest and best if you devote some time to doing problems every day.

Typically the most difficult problems are story problems, since they require some effort before you can begin calculating. Here are some pointers for doing story problems:

1. Carefully read each problem twice before writing anything.
2. Assign letters to quantities that are described only in words; draw a diagram if appropriate.
3. Decide which letters are constants and which are variables. A letter stands for a constant if its value remains the same throughout the problem.
4. Using mathematical notation, write down what you know and then write down what you want to find.
5. Decide what category of problem it is (this might be obvious if the problem comes at the end of a particular chapter, but will not necessarily be so obvious if it comes on an exam covering several chapters).
6. Double check each step as you go along; don’t wait until the end to check your work.
7. Use common sense; if an answer is out of the range of practical possibilities, then check your work to see where you went wrong.
Suggestions for Using This Text

1. Read the example problems carefully, filling in any steps that are left out (ask someone for help if you can’t follow the solution to a worked example).

2. Later use the worked examples to study by covering the solutions, and seeing if you can solve the problems on your own.

3. Most exercises have answers in Appendix A; the availability of an answer is marked by “⇒” at the end of the exercise. Clicking on the arrow will take you to the answer. The answers should be used only as a final check on your work, not as a crutch. Keep in mind that sometimes an answer could be expressed in various ways that are algebraically equivalent, so don’t assume that your answer is wrong just because it doesn’t have exactly the same form as the given answer.

4. A few figures in the pdf and print versions of the book are marked with “(AP)” at the end of the caption. Clicking on this in the pdf should open a related interactive applet or Sage worksheet in your web browser. Occasionally another link will do the same thing, like this example. (Note to users of a printed text: the words “this example” in the pdf file are blue, and are a link to a Sage worksheet.) In the html version of the text, these features appear in the text itself.