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# Single Variable Calculus

*Early Transcendentals*

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This text was initially written by David Guichard. The single variable material in chapters 1–9 is a modification and expansion of notes written by Neal Koblitz at the University of Washington, who generously gave permission to use, modify, and distribute his work. New material has been added, and old material has been modified, so some portions now bear little resemblance to the original.

The book includes some exercises and examples from *Elementary Calculus: An Approach Using Infinitesimals*, by H. Jerome Keisler, available at <http://www.math.wisc.edu/~keisler/calc.html> under a Creative Commons license. In addition, the chapter on differential equations (in the multivariable version) and the section on numerical integration are largely derived from the corresponding portions of Keisler's book.

Some exercises are from the OpenStax Calculus books, available free at <https://openstax.org/subjects/math>.

Albert Schueller, Barry Balof, and Mike Wills have contributed additional material.

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The current version of the text is available at <https://www.whitman.edu/mathematics/multivariable/>.

I will be glad to receive corrections and suggestions for improvement at [guichard@whitman.edu](mailto:guichard@whitman.edu).

*For Kathleen,  
without whose encouragement  
this book would not have  
been written.*

# Contents

## **1**

### Analytic Geometry 13

- 1.1 Lines . . . . . 14
- 1.2 Distance Between Two Points; Circles . . . . . 19
- 1.3 Functions . . . . . 20
- 1.4 Shifts and Dilations . . . . . 25

## **2**

### Instantaneous Rate of Change: The Derivative 29

- 2.1 The slope of a function . . . . . 29
- 2.2 An example . . . . . 34
- 2.3 Limits . . . . . 36
- 2.4 The Derivative Function . . . . . 46
- 2.5 Properties of Functions . . . . . 51

5

Contents 7

## **6**

### Applications of the Derivative 117

- 6.1 Optimization . . . . . 117
- 6.2 Related Rates . . . . . 129
- 6.3 Newton's Method . . . . . 137
- 6.4 Linear Approximations . . . . . 141
- 6.5 The Mean Value Theorem . . . . . 143

## **7**

### Integration 147

- 7.1 Two examples . . . . . 147
- 7.2 The Fundamental Theorem of Calculus . . . . . 151
- 7.3 Some Properties of Integrals . . . . . 158

## **8**

### Techniques of Integration 163

- 8.1 Substitution . . . . . 164
- 8.2 Powers of sine and cosine . . . . . 169
- 8.3 Trigonometric Substitutions . . . . . 171
- 8.4 Integration by Parts . . . . . 174
- 8.5 Rational Functions . . . . . 178
- 8.6 Numerical Integration . . . . . 182
- 8.7 Additional exercises . . . . . 187

6 Contents

## **3**

### Rules for Finding Derivatives 55

- 3.1 The Power Rule . . . . . 55
- 3.2 Linearity of the Derivative . . . . . 58
- 3.3 The Product Rule . . . . . 60
- 3.4 The Quotient Rule . . . . . 62
- 3.5 The Chain Rule . . . . . 65

## **4**

### Transcendental Functions 71

- 4.1 Trigonometric Functions . . . . . 71
- 4.2 The Derivative of  $\sin x$  . . . . . 74
- 4.3 A hard limit . . . . . 75
- 4.4 The Derivative of  $\sin x$ , continued . . . . . 78
- 4.5 Derivatives of the Trigonometric Functions . . . . . 79
- 4.6 Exponential and Logarithmic functions . . . . . 80
- 4.7 Derivatives of the exponential and logarithmic functions . . . . . 82
- 4.8 Implicit Differentiation . . . . . 87
- 4.9 Inverse Trigonometric Functions . . . . . 92
- 4.10 Limits revisited . . . . . 95
- 4.11 Hyperbolic Functions . . . . . 100

## **5**

### Curve Sketching 105

- 5.1 Maxima and Minima . . . . . 105
- 5.2 The first derivative test . . . . . 109
- 5.3 The second derivative test . . . . . 111
- 5.4 Concavity and inflection points . . . . . 112
- 5.5 Asymptotes and Other Things to Look For . . . . . 114

8 Contents

## **9**

### Applications of Integration 189

- 9.1 Area between curves . . . . . 189
- 9.2 Distance, Velocity, Acceleration . . . . . 194
- 9.3 Volume . . . . . 197
- 9.4 Average value of a function . . . . . 204
- 9.5 Work . . . . . 207
- 9.6 Center of Mass . . . . . 211
- 9.7 Kinetic energy; improper integrals . . . . . 216
- 9.8 Probability . . . . . 220
- 9.9 Arc Length . . . . . 230
- 9.10 Surface Area . . . . . 232

## **10**

### Polar Coordinates, Parametric Equations 237

- 10.1 Polar Coordinates . . . . . 237
- 10.2 Slopes in polar coordinates . . . . . 241
- 10.3 Areas in polar coordinates . . . . . 243
- 10.4 Parametric Equations . . . . . 246
- 10.5 Calculus with Parametric Equations . . . . . 249

**11**

**Sequences and Series 253**

- 11.1 Sequences . . . . . 254
- 11.2 Series . . . . . 260
- 11.3 The Integral Test . . . . . 264
- 11.4 Alternating Series . . . . . 269
- 11.5 Comparison Tests . . . . . 271
- 11.6 Absolute Convergence . . . . . 274
- 11.7 The Ratio and Root Tests . . . . . 275
- 11.8 Power Series . . . . . 278
- 11.9 Calculus with Power Series . . . . . 281
- 11.10 Taylor Series . . . . . 283
- 11.11 Taylor’s Theorem . . . . . 286
- 11.12 Additional exercises . . . . . 292

**A**

**Selected Answers 295**

**B**

**Useful Formulas 313**

**Index 317**

# Introduction

The emphasis in this course is on problems—doing calculations and story problems. To master problem solving one needs a tremendous amount of practice doing problems. The more problems you do the better you will be at doing them, as patterns will start to emerge in both the problems and in successful approaches to them. You will learn fastest and best if you devote some time to doing problems every day.

Typically the most difficult problems are story problems, since they require some effort before you can begin calculating. Here are some pointers for doing story problems:

1. Carefully read each problem twice before writing anything.
2. Assign letters to quantities that are described only in words; draw a diagram if appropriate.
3. Decide which letters are constants and which are variables. A letter stands for a constant if its value remains the same throughout the problem.
4. Using mathematical notation, write down what you know and then write down what you want to find.
5. Decide what category of problem it is (this might be obvious if the problem comes at the end of a particular chapter, but will not necessarily be so obvious if it comes on an exam covering several chapters).
6. Double check each step as you go along; don’t wait until the end to check your work.
7. Use common sense; if an answer is out of the range of practical possibilities, then check your work to see where you went wrong.

**12 Introduction**

**Suggestions for Using This Text**

1. Read the example problems carefully, filling in any steps that are left out (ask someone for help if you can’t follow the solution to a worked example).
2. Later use the worked examples to study by covering the solutions, and seeing if you can solve the problems on your own.
3. Most exercises have answers in Appendix A; the availability of an answer is marked by “ $\Rightarrow$ ” at the end of the exercise. Clicking on the arrow will take you to the answer. The answers should be used only as a final check on your work, not as a crutch. Keep in mind that sometimes an answer could be expressed in various ways that are algebraically equivalent, so don’t assume that your answer is wrong just because it doesn’t have exactly the same form as the given answer.
4. A few figures in the pdf and print versions of the book are marked with “(AP)” at the end of the caption. Clicking on this in the pdf should open a related interactive applet or Sage worksheet in your web browser. Occasionally another link will do the same thing, like this example. (Note to users of a printed text: the words “this example” in the pdf file are blue, and are a link to a Sage worksheet.) In the html version of the text, these features appear in the text itself.