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This text was initially written by David Guichard. The single variable material in chapters 1–9 is a modification and expansion of notes written by Neal Koblitz at the University of Washington, who generously gave permission to use, modify, and distribute his work. New material has been added, and old material has been modified, so some portions now bear little resemblance to the original.

The book includes some exercises and examples from Elementary Calculus: An Approach Using Infinitesimals, by H. Jerome Keisler, available at http://www.math.wisc.edu/~keisler/calc.html under a Creative Commons license. In addition, the chapter on differential equations (in the multivariable version) and the section on numerical integration are largely derived from the corresponding portions of Keisler’s book. Albert Schueller, Barry Balof, and Mike Wills have contributed additional material.

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I will be glad to receive corrections and suggestions for improvement at guichard@whitman.edu.
For Kathleen, 
without whose encouragement 
this book would not have 
been written.
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The emphasis in this course is on problems—doing calculations and story problems. To master problem solving one needs a tremendous amount of practice doing problems. The more problems you do the better you will be at doing them, as patterns will start to emerge in both the problems and in successful approaches to them. You will learn fastest and best if you devote some time to doing problems every day.

Typically the most difficult problems are story problems, since they require some effort before you can begin calculating. Here are some pointers for doing story problems:

1. Carefully read each problem twice before writing anything.
2. Assign letters to quantities that are described only in words; draw a diagram if appropriate.
3. Decide which letters are constants and which are variables. A letter stands for a constant if its value remains the same throughout the problem.
4. Using mathematical notation, write down what you know and then write down what you want to find.
5. Decide what category of problem it is (this might be obvious if the problem comes at the end of a particular chapter, but will not necessarily be so obvious if it comes on an exam covering several chapters).
6. Double check each step as you go along; don’t wait until the end to check your work.
7. Use common sense; if an answer is out of the range of practical possibilities, then check your work to see where you went wrong.