
Single Variable Calculus

Early Transcendentals



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This text was initially written by David Guichard. The single variable material in chapters 1–9 is a modification and expansion of notes written by Neal Koblitz at the University of Washington, who generously gave permission to use, modify, and distribute his work. New material has been added, and old material has been modified, so some portions now bear little resemblance to the original.

The book includes some exercises and examples from *Elementary Calculus: An Approach Using Infinitesimals*, by H. Jerome Keisler, available at <http://www.math.wisc.edu/~keisler/calc.html> under a Creative Commons license. In addition, the chapter on differential equations (in the multivariable version) and the section on numerical integration are largely derived from the corresponding portions of Keisler's book.

Some exercises are from the OpenStax Calculus books, available free at <https://openstax.org/subjects/math>.

Albert Schueller, Barry Balof, and Mike Wills have contributed additional material.

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The current version of the text is available at <https://www.whitman.edu/mathematics/multivariable/>.

I will be glad to receive corrections and suggestions for improvement at guichard@whitman.edu.

*For Kathleen,
without whose encouragement
this book would not have
been written.*

Contents

<hr/>	
1	
Analytic Geometry	13
1.1 Lines	14
1.2 Distance Between Two Points; Circles	19
1.3 Functions	20
1.4 Shifts and Dilations	25
<hr/>	
2	
Instantaneous Rate of Change: The Derivative	29
2.1 The slope of a function	29
2.2 An example	34
2.3 Limits	36
2.4 The Derivative Function	46
2.5 Properties of Functions	51

6 Contents

<hr/>	
3	
Rules for Finding Derivatives	55
3.1 The Power Rule	55
3.2 Linearity of the Derivative	58
3.3 The Product Rule	60
3.4 The Quotient Rule	62
3.5 The Chain Rule	65
<hr/>	
4	
Transcendental Functions	71
4.1 Trigonometric Functions	71
4.2 The Derivative of $\sin x$	74
4.3 A hard limit	75
4.4 The Derivative of $\sin x$, continued	78
4.5 Derivatives of the Trigonometric Functions	79
4.6 Exponential and Logarithmic functions	80
4.7 Derivatives of the exponential and logarithmic functions	82
4.8 Implicit Differentiation	87
4.9 Inverse Trigonometric Functions	92
4.10 Limits revisited	95
4.11 Hyperbolic Functions	100
<hr/>	
5	
Curve Sketching	105
5.1 Maxima and Minima	105
5.2 The first derivative test	109
5.3 The second derivative test	111
5.4 Concavity and inflection points	112
5.5 Asymptotes and Other Things to Look For	114

6

Applications of the Derivative	117
6.1 Optimization	117
6.2 Related Rates	129
6.3 Newton's Method	137
6.4 Linear Approximations	141
6.5 The Mean Value Theorem	143

7

Integration	147
7.1 Two examples	147
7.2 The Fundamental Theorem of Calculus	151
7.3 Some Properties of Integrals	158

8

Techniques of Integration	163
8.1 Substitution	164
8.2 Powers of sine and cosine	169
8.3 Trigonometric Substitutions	171
8.4 Integration by Parts	174
8.5 Rational Functions	178
8.6 Numerical Integration	182
8.7 Additional exercises	187

9

Applications of Integration	189
9.1 Area between curves	189
9.2 Distance, Velocity, Acceleration	194
9.3 Volume	197
9.4 Average value of a function	204
9.5 Work	207
9.6 Center of Mass	211
9.7 Kinetic energy; improper integrals	216
9.8 Probability	220
9.9 Arc Length	230
9.10 Surface Area	232

10

Polar Coordinates, Parametric Equations	237
10.1 Polar Coordinates	237
10.2 Slopes in polar coordinates	241
10.3 Areas in polar coordinates	243
10.4 Parametric Equations	246
10.5 Calculus with Parametric Equations	249

11

Sequences and Series	253
11.1 Sequences	254
11.2 Series	260
11.3 The Integral Test	264
11.4 Alternating Series	269
11.5 Comparison Tests	271
11.6 Absolute Convergence	274
11.7 The Ratio and Root Tests	275
11.8 Power Series	278
11.9 Calculus with Power Series	281
11.10 Taylor Series	283
11.11 Taylor's Theorem	286
11.12 Additional exercises	292

A

Selected Answers	295
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B

Useful Formulas	313
------------------------	------------

Index	317
--------------	------------

Introduction

The emphasis in this course is on problems—doing calculations and story problems. To master problem solving one needs a tremendous amount of practice doing problems. The more problems you do the better you will be at doing them, as patterns will start to emerge in both the problems and in successful approaches to them. You will learn fastest and best if you devote some time to doing problems every day.

Typically the most difficult problems are story problems, since they require some effort before you can begin calculating. Here are some pointers for doing story problems:

1. Carefully read each problem twice before writing anything.
2. Assign letters to quantities that are described only in words; draw a diagram if appropriate.
3. Decide which letters are constants and which are variables. A letter stands for a constant if its value remains the same throughout the problem.
4. Using mathematical notation, write down what you know and then write down what you want to find.
5. Decide what category of problem it is (this might be obvious if the problem comes at the end of a particular chapter, but will not necessarily be so obvious if it comes on an exam covering several chapters).
6. Double check each step as you go along; don't wait until the end to check your work.
7. Use common sense; if an answer is out of the range of practical possibilities, then check your work to see where you went wrong.

12 Introduction

Suggestions for Using This Text

1. Read the example problems carefully, filling in any steps that are left out (ask someone for help if you can't follow the solution to a worked example).
2. Later use the worked examples to study by covering the solutions, and seeing if you can solve the problems on your own.
3. Most exercises have answers in Appendix A; the availability of an answer is marked by “ \Rightarrow ” at the end of the exercise. Clicking on the arrow will take you to the answer. The answers should be used only as a final check on your work, not as a crutch. Keep in mind that sometimes an answer could be expressed in various ways that are algebraically equivalent, so don't assume that your answer is wrong just because it doesn't have exactly the same form as the given answer.
4. A few figures in the pdf and print versions of the book are marked with “(AP)” at the end of the caption. Clicking on this in the pdf should open a related interactive applet or Sage worksheet in your web browser. Occasionally another link will do the same thing, like this example. (Note to users of a printed text: the words “this example” in the pdf file are blue, and are a link to a Sage worksheet.) In the html version of the text, these features appear in the text itself.