By November 1, 2024, you should submit a written report to the Office of the Provost and Dean of the Faculty, c/o Mary Raschko. Funding for this PIG will only be available through December 1, 2024, so if you are unable to submit the report on time, you should promptly contact Mary. The project report (to be posted on the Center for Teaching and Learning website) should address the following questions:

TO: Mary Raschko
FROM: Kurt Hoffman

Final report of work completed under the support from a PIG.

Project Title: Physics Advanced Laboratory Project Development

The Motivation for this project stems from a need identified by our alumni and an external review team to make our advanced lab course more relevant for the careers our students will eventually pursue after graduation. With that in mind I proposed the development of several laboratory activities to replace existing projects with activities that focused on experimental techniques, exploring the connection between theory and experiment, and an introduction to modeling as a way of assessing experimental results.

- What was the outcome of your project and how did it align with your original goals as stated in your application?

The outcomes of this project will be three multi-week advanced laboratory experiments that will be available for implementation this fall in an independent study course and next spring in the Advanced Laboratory course. Each of the three experiments were developed in close collaboration with one summer research student. The students were given some starting materials and guidance from me and then worked through the development of optimal strategies for setting up the measurements. The research students also worked through the theoretical analysis and modeling, if needed, to make the connection between fundamental physics principles and the complex system they were studying. Finally, the students created drafts of the experimental guidelines they felt were necessary for students to complete the experiments in a reasonable amount of time. I am still in the process of editing these documents to create working laboratory instructions. I have appended pdf copies of the three documents in their current state. One is essentially complete. The other two are in various stages of refinement as I make choices about the ordering of activities and the depth of guidance I will provide in the instructions.

In addition, the students were able to accomplish the stated research goals of the project:

1. Inexpensive ESPI system that we can use in our instructional labs.
2. Python based FFT program that will have wide use in our laboratory research. All three of these laboratory units exploit this technique that avoids the purchasing of a high cost FFT device.

3. Two of the laboratories have Jupyter notebook based measurement instructions. This approach will make it simpler to share programs between students and possibly point the way toward Jupyter notebook based laboratory manuals.

- What sense do you have at this point that your project has enhanced student learning and what will you be looking at in the future to know whether the project was successful?

At this point my students have successfully completed all of the varied measurements we might pursue for two of the experiments. Note, the experiments include options for students in the class to choose from. The research students have completed many more experiments than any classroom lab group would undertake. The key outcomes of this summer's work is to identify the needed support to complete sophisticated data analysis, the guidance required to use more complicated data collection techniques, and clarifying the level of sophistication that seems appropriate for an instructional laboratory experience at the 300 level. The laboratory activities are new, more challenging that previous experiments, and will hopefully stimulate student interest in devising experimental procedures on their own.

- What were the limitations or failings of this project, and how, in retrospect, might they have been better addressed or remedied?

I don't think there are any failings in this project as proposed and subsequently implemented. There are several limitations to this project related to the ambitious goals of the project. The first limitation has been the challenges of upgrading software to make the interfacing with instrumentation straightforward. We have been working through that challenge, but it is taking more time than anticipated. In addition, the move to python based programming has gone better than expected due to my students having learned python in previous computer and physics courses. Still, despite collecting some of the data with python based programs, the debugging and implementation will take more time. Part of this challenge is that I, too, am new to programming with python and using the Jupyter python environment specifically. If I was in a stronger position to suggest coding approaches and better at debugging, we might have made quicker progress.

● How do you envision sharing the results of your work with other colleagues at Whitman (or elsewhere)?

The primary use of these materials will be in the Advanced Laboratory course taught by Prof. Aragoneses during the spring semester of 2025. In addition, we may find that some of the experiments might be useable in the phys255 laboratory course because several of the experiments are acoustical analogs of quantum situations. Hence, some of the experiments can play a role in helping students understand the mathematical results in a tangible system that can help them extend the concepts to the more abstract quantum world. I proposed an NSF grant to make more sophisticated experiments connecting acoustics and quantum physics that was well received despite not getting funded. The basic idea of using acoustics to help students develop mathematical skills and intuition related to quantum analogs was seen as a particularly noteworthy strength of my NSF proposal.

# Motivation

The vibrating string is the classic system for introducing the techniques for solving the wave equation in a one dimensional system. You were given the solutions to this problem in your general physics course without going through the task of setting up and solving the differential equation. Interestingly, we can contrive more complicated string systems that also can be solved with the same techniques, but with slightly more complicated boundary conditions. The goal of this set of experiments is to answer questions such as: "How does a real string motion deviate from the ideal string solutions we studies in general physics?" and "What happens to the normal mode vibrations of a string if it has a discontinuous change in its mass density?".

# Learning Goals

- Verify/Correct the theoretical derivations in a journal article.
- Solve differential equations describing the motion of strings by properly applying boundary conditions to find standing wave frequencies and waveforms.
- Design, implement, and troubleshoot a measurement to verify the article's results.
- Use Fast Fourier Transform (FFT) techniques for data collection.
- Identify new research questions based on your results.

# Introduction

The experiments here are motivated by the following questions:

- Are the frequencies of a uniform string's high harmonic standing waves really integer multiples of the fundamental?
- What are the vibration frequencies of the standing waves on a non-uniform string?

# Review

Newton's 2nd Law can be used to derive the following wave equation for a uniform string of linear mass density $\mu$ under a tension force $T$ (equation (2) from journal article 1):

$$T\frac{\partial^2}{\partial x^2}y(x,t) = \mu\frac{\partial^2}{\partial t^2}y(x,t) \tag{1}$$

The general solution to the wave equation consists of left and right-going traveling waves on an infinitely long string. We can find specific harmonic solutions (standing waves) for a string of length $l$ by imposing boundary conditions on the general solution. Firstly, to ensure that the solution is a harmonic solution, we demand that $y(x,t)$ be a separable equation, meaning that it can be expressed as the product of a time-dependent and position-dependent function. Additionally, we demand that the time-dependent function be $e^{-i\omega t}$.

$$y(x,t) = f(t)y(x) \tag{2}$$

$$y(x,t) = e^{-i\omega t}y(x) \tag{3}$$

In the Experiment 1 pre-lab, you will show the following:

$$y(x) = ae^{ikx} + be^{-ikx} \tag{4}$$

Additionally, for a string of length $l$ that's fixed at both ends, we can demand that $y(0,t) = 0$ and $y(l,t) = 0$. An infinite number of discrete functions satisfy this requirement and form the standing waves you studied in General Physics. The complete set of functions are

$$y(x) = A\sin(\pi nx/l) \tag{5}$$

where $A$ is the amplitude of the wave, and $n$ is a positive integer. Such waves have wavelengths of $\lambda = 2l/n$ and frequencies of:

$$f_n = \frac{n}{2l}\sqrt{\frac{T}{\mu}} \tag{6}$$

This leads to the following relationship for the ideal string that you have previously encountered in your physics coursework:

$$f_n = nf_1 \tag{7}$$

# Experiment 1 Pre-lab

1. Read Experiment 1 and Measurement 1 from this document
2. Read journal article (1). Pay specific attention to Pages 1 and 2 as well as figures 2 and 3 at the bottom of the 4th page
3. Plug equation (3) into equation (1) to obtain an second order ordinary differential equation for $y(x)$. Prove equation (4) is the general solution to that differential equation (show $e^{ikx}$ and $e^{-ikx}$ are solutions to the differential equation).

   *Remember that $v = \sqrt{T/\mu}$ and $v = \omega/k$ where $v$ is the speed of the wave, $\omega = 2\pi f$, and $k = 2\pi/\lambda$.*

4. Verify that when you apply the boundary conditions $y(0) = y(l) = 0$ to equation (4) you get equation (5).

   *Useful Identity:* $\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$

5. Prove that for equation (5), equations (6) and (7) are true. You may find the wave identities from part 3 of the pre-lab useful.

# Experiment 1: Stiff Strings

The derivation of the equation of motion for the ideal string you completed as part of the pre-lab assumes that the tension is the only restoring force acting on the string. However, this is not a complete description of the forces acting for real strings, particularly those under low tension. In the case of a real metal string, there is also a restoring force due to the mechanical properties of the string itself. (Think of a thin guitar string and increase its radius until it becomes a rod that will vibrate even if no tension is acting.) To account for stiffness, the wave equation becomes

$$\mu\frac{\partial^2}{\partial t^2}y(x,t) = T\frac{\partial^2}{\partial x^2}(x,t) - YSk^2\frac{\partial^4}{\partial x^4}y(x,t) \tag{8}$$

where $Y$ is Young's modulus, $S$ is the cross-sectional area, and $k$ is the radius of gyration [2]. With this new stiffness term, the relationship between the harmonics if the string is pinned at both ends is

$$f_n = nf_0\sqrt{1 + Bn^2} \tag{9}$$

where $B = \frac{\pi^2 YSk^2}{Tl^2}$. $f_0$ is the fundamental frequency of the string *without* the stiffness term.

$$f_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \tag{10}$$

For pinned strings $y = \frac{\partial^2}{\partial x^2}y = 0$ at the pinned end(s) independent of time [2].

### Measurement 1

Set up a sonometer. Use a lever and weights to apply a known tension force to the string. Use bridges to satisfy the pinned boundary conditions. Record the length and tension of the string.

*Torque Review:* $\tau = F \cdot r$ & $\tau = I\alpha$

Ensure that the tension in the string is high enough that the pitch of the sonometer is audibly stable. Set up the bridges so the length of the string between the bridges is most of the length of the sonometer. For thicker strings, you may consider cutting grooves into the bridges to ensure that the strings do not move horizontally.

Pluck the string and make an FFT measurement on the string. From our experience performing this experiment, we have found it ideal when $100Hz < f_1 < 200Hz$.

1. Identify at least ten harmonics with at least one harmonic where $n \geq 20$. Ensure that the spacing between the chosen harmonics is similar.

   *Eg. Pick harmonics like n = 1, 3, 5, 7, 9, 11, 13, 15, 17, 20. Do not pick harmonics like n = 1, 2, 3, 4, 5, 6, 7, 8, 9, 20.*

   To identify harmonics, use the Voltage vs. frequency plot ($V$ vs. $f$) generated by the FFT to approximate the various $f_n$s. Then, fit the data in that region around each $f_n$ with the following Gaussian distribution.

   $$V = \gamma e^{-0.5(\frac{f-f_n}{\sigma})} \tag{11}$$

   $\sigma$ is the standard deviation for that $f_n$ and $\gamma$ is just a scaling constant.

2. Fit your $f_n$s with equation (9) to obtain the ideal fundamental frequency of the string without stiffness ($f_0$) and the stiffness constant ($B$). Graph the difference between your model and your measured data to ensure that your model accurately describes your data.

3. Graph the difference between your measured data and the elementary physics model given by equation (7). Use your modeled value of $f_0$ for $f_1$ as that is the ideal fundamental frequency if there were no stiffness. Under what circumstances can equation (7) approximately model a uniform string?

4. Calculate the linear mass density ($\mu$) of the string.

   *You cannot use equation* (6) *as all your measured data points include stiffness.*

## Experiment 2 Pre-lab

**Note: journal articles 1 and 2 use different variable names.**

1. Read journal article 2. Note the similarities in the derivations of equations (4) and (5) from journal article 2 to the pre-lab to Experiment 1. Verify that equation (7) from the article under the boundary conditions in equations (8a), (8b), (8c), and (8d) from the article results in equation (12) from *THIS* document.

   *Useful Identities:* $\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$ & $\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$

2. Equation (9) from journal article 2 is incorrect. Use the result from part 1 of this pre-lab to fix it.

# Experiment 2: Compound Strings

Under the boundary conditions given by journal article 2, we can derive the following relationships if we assume $y(0) \neq 0$ [1] [1].

$$k_1 \cot(k_1 l_1) + k_2 \cot(k_2 l_2) = 0 \tag{12}$$

$$\sqrt{\rho_1} \cot\left[(2\pi l_1 \sqrt{\rho_1/T}) f\right] + \sqrt{\rho_2} \cot\left[(2\pi l_2 \sqrt{\rho_2/T}) f\right] = 0 \tag{13}$$

To determine frequencies of the wave's harmonics, we have to solve for $f$ through non-algebraic methods. $f_n$ is the $n$th smallest positive value for $f$ for which equation (13) is true.

### Measurement 2

Set up the sonometer similarly to Measurement 1 but with the compound string. Take note of $T$, $l_1$, and $l_2$. Attached is a link to a program on Desmos Graphing Calculator that may prove useful for this measurement.

`https://www.desmos.com/calculator/qtxfbiaegi`

1. Use the techniques from Measurement 1 to calculate $\rho_1$ and $\rho_2$. You do not need to take as many data points as in Measurement 1.
2. Calculate the first 5 (positive) solutions to equation (13) given your $T$, $\rho_1$, $\rho_2$, $l_1$ and $l_2$. You can use the Desmos graph for this purpose. The zeroes of that graph given *your* experimental constants $T$, $\rho_1$, $\rho_2$, $l_1$ and $l_2$, are the allowed $f$s.
3. Perform an FFT measurement on the compound string. Identify the first five harmonics in your data. How does your recorded data compare with the solutions given by your model.

   Don't worry if the modeled harmonics are slightly out of your data's error range. Small errors in $\rho$ or $l$ can alter the model, and there's no convenient way to take all of those errors into account.
4. Can you think of any problems/deficiencies with the model?

# Possible Extensions

1. Use the models and methods in either Measurements 1 or 2 and alter one of the parameters ($\rho$, $T$, $l$, *etc.*). How does altering these parameters affect the model?
2. Extend Measurement 1 by choosing a different monofilament string to determine the B parameter for a different string manufacturer or string material. Options include guitar, violin, viola, or cello strings.
3. Repeat Measurement 1, but clamp the strings at both ends instead of pinning them. For clamped strings $y = \frac{\partial}{\partial x} y = 0$ at the clamped ends and $f_n \approx n f_0[1 + \beta + \beta^2(1 + \frac{n^2\pi^2}{8})]$ where $\beta = \frac{2k\sqrt{YS}}{l\sqrt{T}}$. [3]
4. Repeat Measurement 1, but use an over-wound string if your initial measurements weren't made on an over-wound string or vice versa. Use the same $T$ and $l$. How is $B$ affected by whether the string is or isn't over-wound?

[1] This is a valid assumption if $\frac{l_1\sqrt{\rho_1}}{l_2\sqrt{\rho_2}}$ is irrational

# References

[1] T. A. Barnebey. Note on "Normal modes of a compound string". *American Journal of Physics*, 44(5):485–486, 05 1976.

[2] Chris J. Murray and Scott B. Whitfield. Inharmonicity in plucked guitar strings. *American Journal of Physics*, 90(7):487–493, 07 2022.

[3] Thomas D. Rossing and Neville H. Fletcher. *Principles of Vibration and Sound*. Springer, 2004.

# Learning Goals

- Understand impedance and reflectivity in acoustics, and apply theoretical equations using python.
- Explore theoretical modeling designed using python as a method to verify experimental results.
- Use Fast Fourier Transform (FFT) techniques for data collection.
- Compare theoretical models with FFT measurements, and verify these results with a journal article.
- Identify new research questions based on your results.

# Introduction

These experiments are motivated by the following questions:

- How does impedance and reflectivity add in a periodic cylindrical lattice, and what effect does this have on the resulting band gap or blocked frequencies?
- What effect does a defect element (e.g. different length of pipe) have on the band-gap pattern in an otherwise periodic cylindrical lattice?
- How do harmonics in an open tube transform into a pattern of blocked frequencies as you increase the number of elements (pipes) in a periodic system?

# Review

You should recall that electrical impedance is defined as the ratio of the ac voltage/current in an electronic circuit. In the world of acoustics, the acoustic impedance is defined as the ratio of pressure difference across a section of pipe over the volume flow rate. The volume flow rate, U, is related to the particle velocity and the cross-sectional area of the pipe; therefore, this equation known as the characteristic impedance for an pipe can be defined as [2]:

$$Z_o = \frac{p(x,t)}{U(x,t)} = \frac{\rho c}{S} \tag{1}$$

Where p is the density of air (give value), c is the speed of sound (give value), and S is the cross sectional area of the pipe. The final ratio is obtained by noting that the particle velocity is proportional to the pressure difference.

In the case of multiple pipes of different radii (cross sectional areas), we can use the reflection coefficient, r, to show how these impedances combine at the junction between the two different pipes. The following equation shows the amplitude reflection coefficient at the junction between two different sized pipes [1]:

$$r_{ij} = \frac{Z_i - Z_j}{Z_i + Z_j} \tag{2}$$

In the case of a second boundary at distance, d, from the first, we need to account for multiple reflections between the two junctions, resulting in an imaginary term added [1]:

$$r_{123} = \frac{r_{12} + r_{23}e^{2ikd}}{1 + r_{12}r_{23}e^{2ikd}} \tag{3}$$

Here, $k = \frac{2\pi}{\lambda}$ and $r_{12}$ and $r_{23}$ refer to the difference in impedance at each junction as seen in the previous reflection coefficient equation.

This pattern of reflection junctions can be applied recursively to any number of boundaries with arbitrary distances. *Refer to reference [1] for a more detailed explanation of this method.* In the case of the theoretical modeling used in the following experiments, a python program was used to write this recursive addition of reflection junctions for any number of elements (pipes) in a system.

# Experiment 1 Pre-Lab

1. Read this entire document and read the journal article [1] and [2]. Focus on the explanations of acoustic impedance and the reflection coefficient equations.

2. Open the attached theoretical model python program on Jupyter Notebook. Read through the code for an understanding of the process behind how the reflection adds with each junction in the system.

   From journal article [2], for a finite pipe of length, L, we have to account for the impedance at the start of the pipe (x=0) and the end (x=L). The resulting acoustic input impedance corresponds to the following equation:

   $$Z_{in} = Z_o \frac{Z_L + iZ_o tan(kL)}{Z_o + iZ_L tan(kL)} \tag{4}$$

   Where $Z_L$ is the impedance at the end of the pipe and $Z_o$ corresponds to the characteristic impedance equation, discussed in the review section. A pipe open at one end has a terminal impedance, $Z_L \approx 0$ and the closed end results in $Z_{in} \approx \infty$. The harmonic frequencies of this closed-open pipe has the following equation [2]:

   $$f_n = n\frac{c}{4L} \text{ (n= 1,3,5...)} \tag{5}$$

3. Using equation (4), and the boundary conditions for a closed-open pipe, discussed above, derive equation (5). *Recall that* $k = \frac{2\pi}{\lambda}$

# Experiment 1: Periodic Cylindrical Lattice

**Measurement 1 (Open Tube Case):**
Open the attached python program used for the set-up of the experiment. Read through the Jupyter Notebook on how the program will automatically start the recording of the microphone, start the speaker, and end the recording once run.

The microphone and speaker both have attached circuits that help to amplify the signal and create a constant volume flow rate for the speaker (allowing the microphone to measure impedance). Use a myDAQ device to connect the microphone to the computer ensuring that you use the a0+ / a0- terminals.

Attach the output of the speaker circuit to a signal generator using banana clips, remember to connect one clip to ground. Use a USB cord to connect the signal generator to the computer.

Connecting the signal generator to the computer:

- Open NI-MAX on the computer. Go to Devices and Instrumentation and find the SDG signal generator (it may be listed under USB devices).
- Click on the self-test button at the top of the panel. If the device is properly connected, you will get a message saying the device is working.
- Click on device details and find the device VISA resource name. It will look similar to this: USB0::0xF4ED::0xEE3A::SDG10GAC3R0011::INSTR
- Copy the VISA resource name and paste it into the corresponding spot on the Jupyter Notebook program
- Select the utility button on the signal generator. Then go to the interface option and ensure that USBTMC is selected.

Test that you have correctly connected the speaker and microphone by running a couple test runs with the program. Insert the speaker and microphone into the 3D printed connection piece.

1. Measure the length of the smallest set of tubes (blue pipes). Insert one end into the other side of the speaker/microphone connection piece.
2. Run the experiment testing program. Save the recording and upload it to the FFT python program. This program will take in the recorded sample and perform a FFT measurement on the sample.
3. You should see peaks at each of the harmonic frequencies. Use the frequency equation found in the prelab to verify the location of the first 5 harmonics.

**Measurement 2 (Simple Periodic System):**

1. Measure the length of two more segments of pipe (one larger and another blue pipe).
2. Attach the pipes to the open tube using pipe connectors. Ensure that all the junctions of pipes are tight (if too loose, use tape to secure the pipes to the connections).
3. In the theoretical modeling program, replace the lengths of the pipe with your measurements. Check to make sure that the pipe radius list is also updated. It should alternate between the smaller radius (blue pipe) and the larger radius (white pipe).
4. Run the theoretical model to see how the periodic system affects the reflections and interference in the tube. Observe how bands of frequencies begin to disappear (blocked) from the model.
5. Run the experiment python program and use the same method as before to obtain a FFT measurement of the recording.
6. Compare the FFT measurement to the theoretical model. Do the blocked frequencies in the model match the FFT measurement? What differences are there between the theoretical and experimental and what could account for these differences?

**Measurement 3 (Building on the Periodic System):**

1. Repeat the same progress used in Measurement 2 for a five, seven, and nine element system. Remember to measure the length of each additional set of pipes and update the theoretical model with these values. You will need to add the radii of the pipes to the list in the theoretical model.
2. Take screenshots of both the FFT measurements and theoretical models for each of the systems you create.
3. How do the results change as the number of elements in the system increases? What effect does the increase of elements have on the resulting band-gap pattern?

# Experiment 2 Pre-Lab

# Experiment 2: Defects in a Periodic Cylindrical Lattice

# Possible Extensions

1. Create a python program that solves for the total impedance instead of the reflectivity of the system. Use these impedance measurements to compare to the FFT experimental results.

# References

[1] W.M. Robertson J.N. Munday C. Brad Bennett. "Band Gaps and Defect Modes in Periodically Structured Waveguides". In: *The Journal of the Acoustical Society of America* 112.1353 (2002). URL: https://pubs.aip.org/asa/jasa/article-abstract/112/4/1353/550123/Band-gaps-and-defect-modes-in-periodically?redirectedFrom=fulltext.

[2] Herbert Jaeger. "Using an Impedance Probe to Teach Musical Acoustics to Non-science Majors". In: *Acoustical Society of America* 42.035009 (2020). URL: https://pubs.aip.org/asa/poma/article/42/1/035009/958722/Using-an-impedance-probe-to-teach-musical.

# Motivation

The motion of two dimensional surfaces play important roles in a wide range of objects, particularly those that make sound. For this activity, the focus will be on membranes and plates. The key difference between these two terms is that an ideal membrane has no stiffness so the motion is determined by an applied tension (drum, eardrum), whereas a plate motion is determined by the internal stiffness of the material (cymbals, bells, chimes). The studies you will complete in this laboratory activity explore the mathematical description of these two types of motion and some techniques for measuring the surface vibrations of these materials. The questions you are asked to consider are: "What are the normal modes of a drumhead and why aren't they musically useful in general?" and "How is a vibrating plate different from a vibrating membrane in terms of describing it's motion?".

# Learning Goals

- Explore the behavior of a two dimensional vibrational system with cylindrical and rectangular symmetry.
- Solve differential equations describing the motion of a surfaces that have restoring forces dominated by (1) tensile stress and (2) internal stiffness.
- Design, implement, and troubleshoot a measurement to verify a research article's results.
- Use Electronic Speckle Pattern Interferometry to measure the normal mode shapes and frequencies of vibrating surfaces.
- Identify new research questions based on your results.

# Introduction

The experiments here are motivated by the following questions:

- What are the normal mode vibrations of rectangular and cylindrical objects.
- How does stiffness differ from tension as a mechanism for restoring forces for vibrating systems?

# Review

The theory describing the vibrations of two dimensional surfaces was worked out in the late 1800s so research work more recently has been focused on modified membranes or numerical modeling of complex materials such as wood. Read this book excerpt for an example derivation of the wave equation for two-dimensional membranes. The wave equation

arises from applying Newton's 2nd Law to a small square of a membrane of mass density *sigma* and tension $T$:

$$\sigma \frac{\partial^2}{\partial t^2} z(x,y,t) = T\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x^2}\right) z(x,y,t) = T\nabla^2 z(x,y,t) \tag{1}$$

The ($\nabla$) operator is the generic expression for the spatial derivative that takes different forms in rectangular and cylindrical coordinates. The speed of the waves is given by $v = \frac{T}{\sigma}$. To find the normal modes of vibration of a drumhead we first write the wave equation in cylindrical coordinates. We will also make an assumption that we can use separation of variables as a method for solving the wave equation and that the time dependence is described by harmonic functions.

$$z(r,\theta,t) = R(r)\Theta(\theta)e^{i\omega t} \tag{2}$$

$$-\omega^2 z(r,\theta,t) = \frac{T}{\sigma}\left(\frac{\partial^2 z(R,\Theta,t)}{\partial r^2} + \frac{1}{r}\frac{\partial z(R,\Theta,t)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 z(R\Theta,t)}{\partial^2 \theta}\right) \tag{3}$$

The textbook reference from above outlines the solutions to this wave equation and introduces the Bessel function, which is likely unfamiliar to you. The general solutions take the form:

$$z(r,\theta,t) = AJ_m(kr)e^{\pm im\theta}e^{i\omega t} \tag{4}$$

The final step is to impose the bpdge of the drum restricted from moving, so z(a,$\theta$,t)=0 at all angles and times.

## Activity 2: The Drum Head

- Create a new project in COMSOL, select 'Model Wizard,' then 3-D. When selecting physics, pick membrane, and for study, pick eigenfrequency.
- You are now ready to begin modeling. Referencing the previous example, try to model the drum head. To start, you will need to know the radius and thickness of the drum head. Also note that the drum head is made of mylar.

### Data Collection

- Now, it is time to collect data and find the resonant frequencies of the drum head. Replace the plate with the drum, pointing the drum head toward the camera. Open NI MAX and click on the camera, under devices. Click on grab, then right click and hit zoom to fit. Make sure that the drum is in the camera's view and also in focus (adjustments can e made with the camera directly). Close NI MAX, as the LabVIEW program will not work if MAX is using the camera.
- In the front panel of the program, you can set a range of frequencies for the program to scroll through. In these settings, set the scan for — frequency range.

- You will need to check that the files from this scan will save in a location that you can access. Click on 'Window' the 'block diagram.' Locate where the files are save, and change it so that the path will save to a desired folder. Next open the folder and start the program. If files begin to save, then you can continue on.
- Turn the lights off and begin the scan. If you wish to leave the room, you can do so. The scan should take around 5 minutes. If you leave the room, do so carefully, as it is dark and you don't want to trip.
- Once you have your data collected, you can scroll through the images. If all went well, you should be able to see resonance patterns scattered throughout the data.

Added Masses

- Add mass to the drum head Once you have saved your data from the previous scan. Use magnets to act as additional mass in the drum head. Once the magnets are in place, begin another scan.
- Once you have this data, save it to your drive.

Data Analysis

- In your data, look for resonance patterns. Next, go to your model in COMSOL. In a data table, align the eigenfrequencies which have corresponding resonance patterns. How accurate is your model? Do the frequencies shift monotonically? Are the modes in the same order? What else do you observe about their differences?
- Next, compare your data of the drum head with a mass on it to the data without a mass on it. How is the data different?

# Prelab

Refer to the book excerpt link above and show the following steps in the derivation of the general solution written in Eq. 4.

- Show that the angular boundary condition applied to the general solutions of the differential equation (3.8) requires m to be an integer.
- Show that the radial equation in (3.8) reduces to the Bessel equation that appears just below that equation in the text.
- The general solutions to the Bessel equation are Bessel functions of the first kind $J_m(kr)$ and Bessel functions of the second kind $\Upsilon_m(kr)$. Look up the propertis of the Bessel functions of the second kind and explain why these functions can be excluded for the drumhead case.

Prelab:

Read this paper until section 3.1. The paper focuses on the vibration of a rectangular membrane. The following equation is the eigenvalue equation given in the paper:

$$\frac{\delta^2 W}{\delta x^2} + \frac{\delta^2 W}{\delta y^2} + \frac{\omega^2}{T}\rho(x,y) = 0 \tag{5}$$

In modeling this membrane, two sets of boundary conditions must be met. This first is given because the membrane is fixed along all its edges:

$$W_1(x,0) = W_2(x,0) = 0 = W_1(x,b) = W_2(x,b) \tag{6}$$

$$W_1(-\alpha a) = 0 = W_2[(1-\alpha)a] \tag{7}$$

The second set of conditions is given to ensure continuity of displacement and slope:

$$W_1(0,y) = W_2(0,y) \tag{8}$$

.

$$W_{1,x}(0,y) = W_{2,x}(0,y) \tag{9}$$

The following equations satisfy these boundary conditions:

$$W_1(x,y) = A\sin\left[k_1(x+\alpha a))\right]\sin\left(\frac{n\pi y}{b}\right) \tag{10}$$

$$W_2(x,y) = B\sin\left[k_2((1-\alpha)a - x)\sin\left(\frac{n\pi y}{b}\right)\right. \tag{11}$$

Using these equations, and guidance from the paper, derive equation 11 from the paper.

## Day 1: The Square Aluminum Plate

Today's lab will focus on modeling the eigenfrequencies of a square aluminum plate, taking measurements to determine the eigenfrequencies, and comparing the model to the data.The goal of today is to learn how to use COMSOL and LabVIEW. The square aluminum plate is fixed at the center and free at the edges. In this particular case, we can only numerically model the eigenfrequencies. COMSOL is a software that will allow us to do so. On the lab computer, find the file labeled '...'This is a pre-made model of the plate you will be taking measurements on today.

- Study 1:Under the Model Builder, click on Study 1. Under the study settings, click 'compute.' After a few moments, COMSOL will have computed the eigenfrequencies of this plate. In 'data' you can scroll through several different modes (you will need to click 'graph' each time you select a different frequency.
- Parameters:In the 'Model Builder', under 'Global Definitions', click on 'Pi.' Experiment with changing the width (h1) of the plate and computing solutions again. Set it back to 1.9 mm once you are done.

- Geometry: Under 'Component 1', expand 'Geometry 1'. You will see several work planes,expand 'Work Plane 1', then 'Plane Geometry'. Click on 'Rectangle 1'. In the settings of 'Rectangle 1', in the size and shape section, you can see how the parameters are used. Next, click on 'Extrude 1,'here you can see how the width of the plate is extruded from the plane. Click on the different work planes to see how they correspond to each component.

- Materials: Expand 'Materials', click on 'Aluminum 3003-H18. In the 'Geometry Entity Selection' under settings, you can see that all domains are selected. This tells COMSOL that all of the components are made of aluminum. if you pull down 'Material Contents' you can see the properties of aluminum that were built into this material in COMSOL.If there are terms here you are unfamiliar with, look them up.

- Solid Mechanics: Expand 'Solid Mechanics' and click on 'Free 1.' Take note of which domains are selected.Next, click 'Prescribed Displacement,' take note of the domains selected and the direction it is in. How do you think these factors play into modeling the plate?

Next, you will use Electronic Speckle Pattern Interferometry (ESPI) to find the resonant frequencies of the square plate.
Electronic Speckle Pattern Interferometry enables us to see the vibrational modes of an object. It utilizes the coherence between two beams of light to create an interference pattern. One beam, called the object beam, causes a phase shift when it moves. When the plate is vibrating at a resonant frequency, we are able to see the mode due to a phase shift. To get a better understanding of how electronic speckle pattern interferometers work, read section 2, theory, from this paper. In our lab, a single laser beam passes through a beam splitter which sends more light through to the object beam. Next, the object beam is reflected off the target and recombined with the reference beam, then the image is captured by the camera. The reference beam travels a distance, equal to that of the object beam, then is recombined with the object beam. Along this path, one of the mirrors is connected to a piezo. This can be driven by a signal generator to cause a phase shift, increasing the number of rings in each resonance pattern and also inverting the final image. The images are collected sequentially, and each new image is subtracted from the previous one, resulting in a resonance pattern.

LabVIEW Tutorial

- 1. Open LabVIEW 32-bit and open the existing file called 'ESPIZWOPiezo...'

- 2. On the front panel of this program, find the control labeled 'camera' and select the appropriate camera.

- 3. Now, look at the controls for exposure time, gain, and final amplification. Start by setting the exposure time to 0.125, the gain to 20, and the final amplification to 40. Then, turn on the low pass filter.

- 4. Also in the front panel, set the amplitude of the piezo to 5 Vpp, and the offset to 5 Vpp. Set the frequency of the piezo to 2 Hz

- 5. Turn on both signal generators, turn off the lights, and high the arrow in the top left corner of the program. It will ask you to label the files for saving. Create an appropriately labeled folder and to save the images in there, and label the file with your name, then 'squareplatescan.'

Data Analysis

- Generate a plot of contrast vs. frequency using the data you collected. Next, plot the modeled eigenfrequencies as vertical lines on the same plot. Are they within the line width of the data you collected?)

## Activity 1: The Drum Head

- Create a new project in COMSOL, select 'Model Wizard,' then 3-D. When selecting physics, pick membrane, and for study, pick eigenfrequency.

- You are now ready to begin modeling. Referencing the previous example, try to model the drum head. To start, you will need to know the radius and thickness of the drum head. Also note that the drum head is made of mylar.

Data Collection

- Now, it is time to collect data and find the resonant frequencies of the drum head. Replace the plate with the drum, pointing the drum head toward the camera. Open NI MAX and click on the camera, under devices. Click on grab, then right click and hit zoom to fit. Make sure that the drum is in the camera's view and also in focus (adjustments can e made with the camera directly). Close NI MAX, as the LabVIEW program will not work if MAX is using the camera.

- In the front panel of the program, you can set a range of frequencies for the program to scroll through. In these settings, set the scan for — frequency range.

- You will need to check that the files from this scan will save in a location that you can access. Click on 'Window' the 'block diagram.' Locate where the files are save, and change it so that the path will save to a desired folder. Next open the folder and start the program. If files begin to save, then you can continue on.

- Turn the lights off and begin the scan. If you wish to leave the room, you can do so. The scan should take around 5 minutes. If you leave the room, do so carefully, as it is dark and you don't want to trip.

- Once you have your data collected, you can scroll through the images. If all went well, you should be able to see resonance patterns scattered throughout the data.

Added Masses

- Add mass to the drum head Once you have saved your data from the previous scan. Use magnets to act as additional mass in the drum head. Once the magnets are in place, begin another scan.

- Once you have this data, save it to your drive.

Data Analysis

- In your data, look for resonance patterns. Next, go to your model in COMSOL. In a data table, align the eigenfrequencies which have corresponding resonance patterns. How accurate is your model? Do the frequencies shift monotonically? Are the modes in the same order? What else do you observe about their differences?

- Next, compare your data of the drum head with a mass on it to the data without a mass on it. How is the data different?