

Math Seniors, Oral Examination Information

When: Oral Exams will be given during the first weekend of the Spring semester (Saturday, January 23rd 2010).

Who: You will be tested by a committee of math faculty determined by the math faculty.

Duration: The exam is one hour in length.

What to Expect: Half of the exam will be a few questions selected from a standard list of topics we feel all math majors should know. The list is included on the next page. When studying for this portion of the exam you may consult past class notes or texts, a standard pre-calculus book, or even online resources like <http://www.mathworld.com>.

The other half of the exam will depend on your major.

Pure majors: will be tested over standard topics in either *Abstract Algebra I (Math 385)* OR *Real Analysis I (Math 455)*. If you have taken both courses, you may choose which course is to be tested. However, you must make that choice known to your advisor in advance.

Applied majors: will be tested over standard topics in *differential equations*.

(details like the exact time of your exam and the identities of the people on your examination committee will be finalized via email)

A few things every math major should know...

1. Prove the Quadratic Formula.

2. Show that:

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

(Hint: If the integrand were $e^{-(x^2+y^2)}$, we could use polar coordinates)

3. Prove the Pythagorean Theorem.

4. Prove that the collection of prime numbers is infinite.

5. Prove that $\sqrt{2}$ is irrational.

6. Know the definition of the derivative and be able to explain its geometric motivation. Use the definition to calculate:

(a) the derivative of the sine function (be sure to review any necessary trig identities and limits)

(b) the derivative of x^n for any nonzero integer exponent

7. Derive Newton's Method for approximating solutions of the equation $f(x) = 0$.

8. Explain the geometric interpretation of the integral and show that the general definition of the definite integral (for continuous functions) can be written as:

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{k=1}^n f\left(a + k \frac{b-a}{n}\right)$$

Use this to calculate the integral of $f(x) = x^2$ from 0 to b , where $b > 0$.

9. Solve a general first-order linear differential equation $y' + p(x)y = q(x)$.

10. Prove a formula for the sum of a convergent geometric series.

11. Prove Euler's Formula, $e^{i\theta} = \cos(\theta) + i \sin(\theta)$

12. Derive the formula for the distance from a point to a plane.

13. Know how to prove a result via the principle of mathematical induction.

14. Prove that the set of real numbers is uncountable.