
Calculus

Early Transcendentals

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This text was initially written by David Guichard. The single variable material in chapters 1–9 is a modification and expansion of notes written by Neal Koblitz at the University of Washington, who generously gave permission to use, modify, and distribute his work. New material has been added, and old material has been modified, so some portions now bear little resemblance to the original.

The book includes some exercises and examples from *Elementary Calculus: An Approach Using Infinitesimals*, by H. Jerome Keisler, available at <http://www.math.wisc.edu/~keisler/calc.html> under a Creative Commons license. In addition, the chapter on differential equations is largely derived from the corresponding chapter in Keisler's book. Albert Schueller, Barry Balof, and Mike Wills have contributed additional material.

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I will be glad to receive corrections and suggestions for improvement at guichard@whitman.edu.

*For Kathleen,
without whose encouragement
this book would not have
been written.*

Contents

1

Analytic Geometry	1
1.1 Lines	2
1.2 Distance Between Two Points; Circles	7
1.3 Functions	8
1.4 Shifts and Dilations	13

2

Instantaneous Rate of Change: The Derivative	17
2.1 The slope of a function	17
2.2 An example	22
2.3 Limits	24
2.4 The Derivative Function	34
2.5 Adjectives For Functions	39

3

Rules for Finding Derivatives	43
3.1 The Power Rule	43
3.2 Linearity of the Derivative	46
3.3 The Product Rule	48
3.4 The Quotient Rule	50
3.5 The Chain Rule	53

4

Transcendental Functions	59
4.1 Trigonometric Functions	59
4.2 The Derivative of $\sin x$	62
4.3 A hard limit	63
4.4 The Derivative of $\sin x$, continued	65
4.5 Derivatives of the Trigonometric Functions	66
4.6 Exponential and Logarithmic functions	68
4.7 Derivatives of the exponential and logarithmic functions	70
4.8 Implicit Differentiation	75
4.9 Inverse Trigonometric Functions	79
4.10 Limits revisited	82
4.11 Hyperbolic Functions	87

5

Curve Sketching	91
5.1 Maxima and Minima	91
5.2 The first derivative test	95
5.3 The second derivative test	96
5.4 Concavity and inflection points	97
5.5 Asymptotes and Other Things to Look For	99

6

Applications of the Derivative 103

6.1	Optimization	103
6.2	Related Rates	115
6.3	Newton's Method	123
6.4	Linear Approximations	127
6.5	The Mean Value Theorem	129

7

Integration 133

7.1	Two examples	133
7.2	The Fundamental Theorem of Calculus	137
7.3	Some Properties of Integrals	144

8

Techniques of Integration 149

8.1	Substitution	150
8.2	Powers of sine and cosine	155
8.3	Trigonometric Substitutions	157
8.4	Integration by Parts	160
8.5	Rational Functions	163
8.6	Additional exercises	168

9

Applications of Integration 169

9.1	Area between curves	169
9.2	Distance, Velocity, Acceleration	174
9.3	Volume	177
9.4	Average value of a function	183
9.5	Work	187
9.6	Center of Mass	191
9.7	Kinetic energy; improper integrals	197
9.8	Probability	201
9.9	Arc Length	210
9.10	Surface Area	212

10

Polar Coordinates, Parametric Equations 217

10.1	Polar Coordinates	217
10.2	Slopes in polar coordinates	221
10.3	Areas in polar coordinates	223
10.4	Parametric Equations	226
10.5	Calculus with Parametric Equations	229

11

Sequences and Series	233
11.1 Sequences	234
11.2 Series	240
11.3 The Integral Test	244
11.4 Alternating Series	249
11.5 Comparison Tests	251
11.6 Absolute Convergence	254
11.7 The Ratio and Root Tests	255
11.8 Power Series	258
11.9 Calculus with Power Series	261
11.10 Taylor Series	262
11.11 Taylor's Theorem	266
11.12 Additional exercises	272

12

Three Dimensions	275
12.1 The Coordinate System	275
12.2 Vectors	278
12.3 The Dot Product	283
12.4 The Cross Product	289
12.5 Lines and Planes	293
12.6 Other Coordinate Systems	299

13

Vector Functions	305
13.1 Space Curves	305
13.2 Calculus with vector functions	307
13.3 Arc length	315
13.4 Motion along a curve	321

14

Partial Differentiation	323
14.1 Functions of Several Variables	323
14.2 Limits and Continuity	327
14.3 Partial Differentiation	331
14.4 The Chain Rule	337
14.5 Directional Derivatives	340
14.6 Higher order derivatives	344
14.7 Maxima and minima	346
14.8 Lagrange Multipliers	351

15

Multiple Integration	357
15.1 Volume and Average Height	357
15.2 Double Integrals in Cylindrical Coordinates	367
15.3 Moment and Center of Mass	371
15.4 Surface Area	373
15.5 Triple Integrals	375
15.6 Cylindrical and Spherical Coordinates	378
15.7 Change of Variables	382

16

Vector Calculus	389
16.1 Vector Fields	389
16.2 Line Integrals	391
16.3 The Fundamental Theorem of Line Integrals	395
16.4 Green's Theorem	398
16.5 Divergence and Curl	403
16.6 Vector Equations of Surfaces	406
16.7 Surface Integrals	412
16.8 Stokes's Theorem	416
16.9 The Divergence Theorem	420

17

Differential Equations	425
17.1 First Order Differential Equations	426
17.2 First Order Homogeneous Linear Equations	430
17.3 First Order Linear Equations	433
17.4 Approximation	435
17.5 Second Order Homogeneous Equations	438
17.6 Second Order Linear Equations	441
17.7 Second Order Linear Equations, take two	446

A

Selected Answers	451
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B

Useful Formulas	477
------------------------	------------

Index	481
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Introduction

The emphasis in this course is on problems—doing calculations and story problems. To master problem solving one needs a tremendous amount of practice doing problems. The more problems you do the better you will be at doing them, as patterns will start to emerge in both the problems and in successful approaches to them. You will learn fastest and best if you devote some time to doing problems every day.

Typically the most difficult problems are story problems, since they require some effort before you can begin calculating. Here are some pointers for doing story problems:

1. Carefully read each problem twice before writing anything.
2. Assign letters to quantities that are described only in words; draw a diagram if appropriate.
3. Decide which letters are constants and which are variables. A letter stands for a constant if its value remains the same throughout the problem.
4. Using mathematical notation, write down what you know and then write down what you want to find.
5. Decide what category of problem it is (this might be obvious if the problem comes at the end of a particular chapter, but will not necessarily be so obvious if it comes on an exam covering several chapters).
6. Double check each step as you go along; don't wait until the end to check your work.
7. Use common sense; if an answer is out of the range of practical possibilities, then check your work to see where you went wrong.

Suggestions for Using This Text

1. Read the example problems carefully, filling in any steps that are left out (ask someone for help if you can't follow the solution to a worked example).
2. Later use the worked examples to study by covering the solutions, and seeing if you can solve the problems on your own.
3. Most exercises have answers in Appendix A; the availability of an answer is marked by “ \Rightarrow ” at the end of the exercise. In the pdf version of the full text, clicking on the arrow will take you to the answer. The answers should be used only as a final check on your work, not as a crutch. Keep in mind that sometimes an answer could be expressed in various ways that are algebraically equivalent, so don't assume that your answer is wrong just because it doesn't have exactly the same form as the answer in the back.
4. A few figures in the book are marked with “(AP)” at the end of the caption. Clicking on this should open a related Java applet or Sage worksheet in your web browser. Occasionally another link will do the same thing, like this example. (Note to users of a printed text: the words “this example” in the pdf file are blue, and are a link to a Sage worksheet.)