

---

# Calculus

---

This work is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike License. To view a copy of this license, visit <http://creativecommons.org/licenses/by-nc-sa/3.0/> or send a letter to Creative Commons, 543 Howard Street, 5th Floor, San Francisco, California, 94105, USA. If you distribute this work or a derivative, include the history of the document.

This text was initially written by David Guichard. The single variable material (not including infinite series) was originally a modification and expansion of notes written by Neal Koblitz at the University of Washington, who generously gave permission to use, modify, and distribute his work. New material has been added, and old material has been modified, so some portions now bear little resemblance to the original.

The book includes some exercises from *Elementary Calculus: An Approach Using Infinitesimals*, by H. Jerome Keisler, available at <http://www.math.wisc.edu/~keisler/calc.html> under a Creative Commons license. Albert Schueller, Barry Balof, and Mike Wills have contributed additional material.

This copy of the text was produced at 9:52 on 11/18/2009.

I will be glad to receive corrections and suggestions for improvement at [guichard@whitman.edu](mailto:guichard@whitman.edu).

# Contents

## 1

<b>Analytic Geometry</b>	<b>1</b>
1.1 Lines	2
1.2 Distance Between Two Points; Circles	7
1.3 Functions	8
1.4 Shifts and Dilations	14

## 2

<b>Instantaneous Rate Of Change: The Derivative</b>	<b>19</b>
2.1 The slope of a function	19
2.2 An example	24
2.3 Limits	26
2.4 The Derivative Function	35
2.5 Adjectives For Functions	40

v

Contents vii

## 6

<b>Applications of the Derivative</b>	<b>105</b>
6.1 Optimization	105
6.2 Related Rates	118
6.3 Newton's Method	127
6.4 Linear Approximations	131
6.5 The Mean Value Theorem	133

## 7

<b>Integration</b>	<b>139</b>
7.1 Two examples	139
7.2 The Fundamental Theorem of Calculus	143
7.3 Some Properties of Integrals	150

## 8

<b>Techniques of Integration</b>	<b>155</b>
8.1 Substitution	156
8.2 Powers of sine and cosine	160
8.3 Trigonometric Substitutions	162
8.4 Integration by Parts	166
8.5 Rational Functions	170
8.6 Additional exercises	176

vi Contents

## 3

<b>Rules For Finding Derivatives</b>	<b>45</b>
3.1 The Power Rule	45
3.2 Linearity of the Derivative	48
3.3 The Product Rule	50
3.4 The Quotient Rule	53
3.5 The Chain Rule	56

## 4

<b>Transcendental Functions</b>	<b>63</b>
4.1 Trigonometric Functions	63
4.2 The Derivative of $\sin x$	66
4.3 A hard limit	67
4.4 The Derivative of $\sin x$ , continued	70
4.5 Derivatives of the Trigonometric Functions	71
4.6 Exponential and Logarithmic functions	72
4.7 Derivatives of the exponential and logarithmic functions	75
4.8 Limits revisited	80
4.9 Implicit Differentiation	84
4.10 Inverse Trigonometric Functions	89

## 5

<b>Curve Sketching</b>	<b>93</b>
5.1 Maxima and Minima	93
5.2 The first derivative test	97
5.3 The second derivative test	99
5.4 Concavity and inflection points	100
5.5 Asymptotes and Other Things to Look For	102

viii Contents

## 9

<b>Applications of Integration</b>	<b>177</b>
9.1 Area between curves	177
9.2 Distance, Velocity, Acceleration	182
9.3 Volume	185
9.4 Average value of a function	192
9.5 Work	195
9.6 Center of Mass	200
9.7 Kinetic energy; improper integrals	205
9.8 Probability	210
9.9 Arc Length	220
9.10 Surface Area	222
9.11 Differential equations	227

## 10

<b>Sequences and Series</b>	<b>233</b>
10.1 Sequences	234
10.2 Series	240
10.3 The Integral Test	244
10.4 Alternating Series	249
10.5 Comparison Tests	251
10.6 Absolute Convergence	254
10.7 The Ratio and Root Tests	256
10.8 Power Series	259
10.9 Calculus with Power Series	261
10.10 Taylor Series	263
10.11 Taylor's Theorem	267
10.12 Additional exercises	271

**A**

**Introduction to Maple** 275

A.1 Getting Started . . . . . 275

A.2 Algebra . . . . . 276

    A.2.1 Numbers . . . . . 276

    A.2.2 Variables and Expressions . . . . . 277

    A.2.3 Evaluation and Substitution . . . . . 279

    A.2.4 Solving Equations . . . . . 280

A.3 Plotting . . . . . 282

A.4 Calculus . . . . . 284

    A.4.1 Limits . . . . . 284

    A.4.2 Differentiation . . . . . 284

    A.4.3 Implicit Differentiation . . . . . 285

    A.4.4 Integration . . . . . 285

A.5 Adding text to a Maple session . . . . . 286

A.6 Printing . . . . . 286

A.7 Saving your work . . . . . 286

A.8 Getting Help . . . . . 287

**B**

**Selected Answers** 289

**Index** 303

# Introduction

The emphasis in this course is on problems—doing calculations and story problems. To master problem solving one needs a tremendous amount of practice doing problems. The more problems you do the better you will be at doing them, as patterns will start to emerge in both the problems and in successful approaches to them. You will learn fastest and best if you devote some time to doing problems every day.

Typically the most difficult problems are story problems, since they require some effort before you can begin calculating. Here are some pointers for doing story problems:

1. Carefully read each problem twice before writing anything.
2. Assign letters to quantities that are described only in words; draw a diagram if appropriate.
3. Decide which letters are constants and which are variables. A letter stands for a constant if its value remains the same throughout the problem.
4. Using mathematical notation, write down what you know and then write down what you want to find.
5. Decide what category of problem it is (this might be obvious if the problem comes at the end of a particular chapter, but will not necessarily be so obvious if it comes on an exam covering several chapters).
6. Double check each step as you go along; don't wait until the end to check your work.

**xii Introduction**

7. Use common sense; if an answer is out of the range of practical possibilities, then check your work to see where you went wrong.

**Suggestions for Using This Text**

1. Read the example problems carefully, filling in any steps that are left out (ask someone if you can't follow the solution to a worked example).
2. Later use the worked examples to study by covering the solutions, and seeing if you can solve the problems on your own.
3. Most exercises have answers in Appendix B; the availability of an answer is marked by “ $\Rightarrow$ ” at the end of the exercise. In the pdf version of the full text, clicking on the arrow will take you to the answer. The answers should be used only as a final check on your work, not as a crutch. Keep in mind that sometimes an answer could be expressed in various ways that are algebraically equivalent, so don't assume that your answer is wrong just because it doesn't have exactly the same form as the answer in the back.
4. A few figures in the book are marked with “(JA)” at the end of the caption. Clicking on this should open a related Java applet in your web browser.

**Some Useful Formulas**

**Algebra**

Remember that the common algebraic operations have **precedences** relative to each other: for example, multiplication and division take precedence over addition and subtraction, but are “tied” with each other. In the case of ties, work left to right. This means, for example, that  $1/2x$  means  $(1/2)x$ : do the division, then the multiplication in left to right order. It sometimes is a good idea to use more parentheses than strictly necessary, for clarity, but it is also a bad idea to use too many parentheses.

Completing the square:  $x^2 + bx + c = (x + \frac{b}{2})^2 - \frac{b^2}{4} + c$ .

Quadratic formula: the roots of  $ax^2 + bx + c$  are  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .

Exponent rules:

$$a^b \cdot a^c = a^{b+c}$$

$$\frac{a^b}{a^c} = a^{b-c}$$

$$(a^b)^c = a^{bc}$$

$$a^{1/b} = \sqrt[b]{a}$$

**Geometry**

Circle: circumference =  $2\pi r$ , area =  $\pi r^2$ .

Sphere: vol =  $4\pi r^3/3$ , surface area =  $4\pi r^2$ .

Cylinder: vol =  $\pi r^2 h$ , lateral area =  $2\pi r h$ , total surface area =  $2\pi r h + 2\pi r^2$ .

Cone: vol =  $\pi r^2 h/3$ , lateral area =  $\pi r\sqrt{r^2 + h^2}$ , total surface area =  $\pi r\sqrt{r^2 + h^2} + \pi r^2$ .

**Analytic geometry**

Point-slope formula for straight line through the point  $(x_0, y_0)$  with slope  $m$ :  $y = y_0 + m(x - x_0)$ .

Circle with radius  $r$  centered at  $(h, k)$ :  $(x - h)^2 + (y - k)^2 = r^2$ .

Ellipse with axes on the  $x$ -axis and  $y$ -axis:  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .

**Trigonometry**

$\sin(\theta) = \text{opposite/hypotenuse}$

$\cos(\theta) = \text{adjacent/hypotenuse}$

$\tan(\theta) = \text{opposite/adjacent}$

$\sec(\theta) = 1/\cos(\theta)$

$\csc(\theta) = 1/\sin(\theta)$

$\cot(\theta) = 1/\tan(\theta)$

$\tan(\theta) = \sin(\theta)/\cos(\theta)$

$\cot(\theta) = \cos(\theta)/\sin(\theta)$

$\sin(\theta) = \cos(\frac{\pi}{2} - \theta)$

$\cos(\theta) = \sin(\frac{\pi}{2} - \theta)$

$\sin(\theta + \pi) = -\sin(\theta)$

$\cos(\theta + \pi) = -\cos(\theta)$

Law of cosines:  $a^2 = b^2 + c^2 - 2bc \cos A$

Law of sines:  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Sine of sum of angles:  $\sin(x + y) = \sin x \cos y + \cos x \sin y$

$\sin^2(\theta)$  and  $\cos^2(\theta)$  formulas:

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\tan^2(\theta) + 1 = \sec^2(\theta)$$

$$\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}$$

$$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$$

Cosine of sum of angles:  $\cos(x + y) = \cos x \cos y - \sin x \sin y$

Tangent of sum of angles:  $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$