Calculus

This text was initially written by David Guichard. The single variable material (not including infinite series) was originally a modification and expansion of notes written by Neal Koblitz at the University of Washington, who generously gave permission to use, modify, and distribute his work. New material has been added, and old material has been modified, so some portions now bear little resemblance to the original.

The book includes some exercises from *Elementary Calculus: An Approach Using Infinitesimals*, by H. Jerome Keisler, available at http://www.math.wisc.edu/~keisler/calc.html under a Creative Commons license. Albert Schueller, Barry Balof, and Mike Wills have contributed additional material.

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I will be glad to receive corrections and suggestions for improvement at guichard@whitman.edu.

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Introduction

The emphasis in this course is on problems—doing calculations and story problems. To master problem solving one needs a tremendous amount of practice doing problems. The more problems you do the better you will be at doing them, as patterns will start to emerge in both the problems and in successful approaches to them. You will learn fastest and best if you devote some time to doing problems every day.

Typically the most difficult problems are story problems, since they require some effort before you can begin calculating. Here are some pointers for doing story problems:

1. Carefully read each problem twice before writing anything.
2. Assign letters to quantities that are described only in words; draw a diagram if appropriate.
3. Decide which letters are constants and which are variables. A letter stands for a constant if its value remains the same throughout the problem.
4. Using mathematical notation, write down what you know and then write down what you want to find.
5. Decide what category of problem it is (this might be obvious if the problem comes at the end of a particular chapter, but will not necessarily be so obvious if it comes on an exam covering several chapters).
6. Double check each step as you go along; don't wait until the end to check your work.

7. Use common sense; if an answer is out of the range of practical possibilities, then check your work to see where you went wrong.

Suggestions for Using This Text

1. Read the example problems carefully, filling in any steps that are left out (ask someone if you can't follow the solution to a worked example).
2. Later use the worked examples to study by covering the solutions, and seeing if you can solve the problems on your own.
3. Most exercises have answers in Appendix B; the availability of an answer is marked by "⇒" at the end of the exercise. In the pdf version of the full text, clicking on the arrow will take you to the answer. The answers should be used only as a final check on your work, not as a crutch. Keep in mind that sometimes an answer could be expressed in various ways that are algebraically equivalent, so don't assume that your answer is wrong just because it doesn't have exactly the same form as the answer in the back.
4. A few figures in the book are marked with "(JA)" at the end of the caption. Clicking on this should open a related Java applet in your web browser.

Some Useful Formulas

Algebra

Remember that the common algebraic operations have **precedences** relative to each other: for example, mulitplication and division take precedence over addition and subtraction, but are "tied" with each other. In the case of ties, work left to right. This means, for example, that $1/2x$ means $(1/2)x$: do the division, then the multiplication in left to right order. It sometimes is a good idea to use more parentheses than strictly necessary, for clarity, but it is also a bad idea to use too many parentheses.

Completing the square: $x^2 + bx + c = (x + \frac{b}{2})^2 - \frac{b^2}{4} + c$.

Quadratic formula: the roots of $ax^2 + bx + c$ are $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Exponent rules:

$$a^b \cdot a^c = a^{b+c}$$
$$\frac{a^b}{a^c} = a^{b-c}$$
$$(a^b)^c = a^{bc}$$
$$a^{1/b} = \sqrt[b]{a}$$

Geometry

Circle: circumference $= 2\pi r$, area $= \pi r^2$.

Sphere: vol $= 4\pi r^3/3$, surface area $= 4\pi r^2$.

Cylinder: vol $= \pi r^2 h$, lateral area $= 2\pi rh$, total surface area $= 2\pi rh + 2\pi r^2$.

Cone: vol $= \pi r^2 h/3$, lateral area $= \pi r\sqrt{r^2+h^2}$, total surface area $= \pi r\sqrt{r^2+h^2} + \pi r^2$.

Analytic geometry

Point-slope formula for straight line through the point (x_0, y_0) with slope m: $y = y_0 + m(x - x_0)$.

Circle with radius r centered at (h,k): $(x-h)^2 + (y-k)^2 = r^2$.

Ellipse with axes on the x-axis and y-axis: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Trigonometry

$\sin(\theta) = \text{opposite}/\text{hypotenuse}$

$\cos(\theta) = \text{adjacent}/\text{hypotenuse}$

$\tan(\theta) = \text{opposite}/\text{adjacent}$

$\sec(\theta) = 1/\cos(\theta)$

$\csc(\theta) = 1/\sin(\theta)$

$\cot(\theta) = 1/\tan(\theta)$

$\tan(\theta) = \sin(\theta)/\cos(\theta)$

$\cot(\theta) = \cos(\theta)/\sin(\theta)$

$\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right)$

$\cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right)$

$\sin(\theta + \pi) = -\sin(\theta)$

$\cos(\theta + \pi) = -\cos(\theta)$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos A$

Law of sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Sine of sum of angles: $\sin(x+y) = \sin x\cos y + \cos x \sin y$

$\sin^2(\theta)$ and $\cos^2(\theta)$ formulas:

$$\begin{aligned}
\sin^2(\theta) + \cos^2(\theta) &= 1 \\
\tan^2(\theta) + 1 &= \sec^2(\theta) \\
\sin^2(\theta) &= \frac{1-\cos(2\theta)}{2} \\
\cos^2(\theta) &= \frac{1+\cos(2\theta)}{2}
\end{aligned}$$

Cosine of sum of angles: $\cos(x+y) = \cos x\cos y - \sin x \sin y$

Tangent of sum of angles: $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$